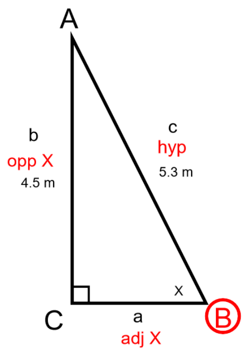
# Trig Equations



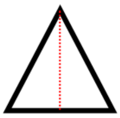

$$\sin X = \frac{opp}{hyp} \qquad \cos X = \frac{adj}{hyp} \qquad \tan X = \frac{opp}{adj}$$

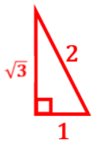
| | | |
|---|---|---|
| $sinX = \frac{4.5}{5.3}$ | $cosX = \frac{a}{5.3}$ | $tanX = \frac{4.5}{a}$ |
| $sinX = 0.8491$ | $cosA =$ | $tanA =$ |
| $sin^{-1}(0.8491)$ | $cos^{-1}(\square) =$ | $tan^{-1}(\square)$ |
| $X = 58°$ | | |

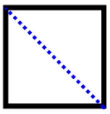
## *Solving Trigonometric Equations*

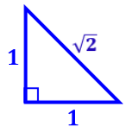
**Outcomes:** Solve trigonometric equations that have exact solutions.

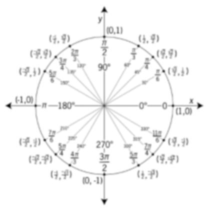
Use triangles or the unit circle to find exact solutions to trigonometric equations:







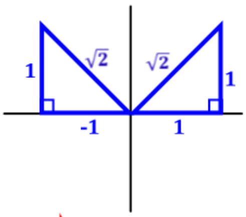



State the exact values of:

| | | |
|---|---|---|
| $\text{Cos } \frac{\pi}{6} = \frac{\sqrt{3}}{2}$ | $\text{Cos } \frac{\pi}{3} = \frac{1}{2}$ | $\text{Cos } \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ |
| $\text{Sin } \frac{\pi}{6} = \frac{1}{2}$ | $\text{Sin } \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ | $\text{Sin } \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ |
| $\text{Tan } \frac{\pi}{6} = \frac{1}{\sqrt{3}}$ | $\text{Tan } \frac{\pi}{3} = \frac{\sqrt{3}}{1}$ | $\text{Tan } \frac{\pi}{4} = \frac{1}{1}$ |

**Examples:**

1. Find the exact value(s) of $\theta$ where $0^0 \le \theta < 360^0$

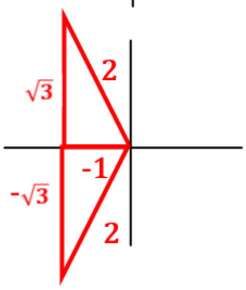



a) $\sin\theta = \frac{1}{\sqrt{2}}$ $\frac{\text{opp}}{\text{hyp}}$

b) $\cos\theta = \frac{\sqrt{2}}{2}$

reference angle = 45

$\theta = 45°, 135°$




c) $\sec\theta = -2$ $\frac{2 = \text{hyp}}{-1 = \text{adj}}$

d) $\cot\theta = 1$

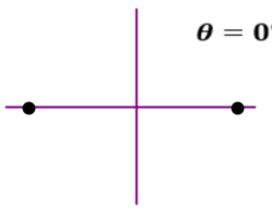
reference angle = 60

$\theta = 120°, 240°$

e) $\sin\theta = 0$ $\frac{\text{opp} = 0}{\text{hyp} = 1}$

f) $\cos\theta = -1$

$\theta = 0°, 180°$



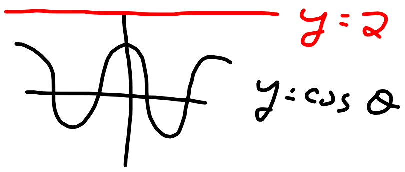
**Trig Equations**

2. Solve the equation $\cos^2 x - \cos x - 2 = 0$ for $0 \le x < 2\pi$. Then write the general solution of the equation.

$$A^2 - A - 2 = 0$$
$$(A-2)(A+1) = 0$$

$$(\cos x - 2)(\cos x + 1) = 0$$

$$\cos x = 2 \qquad \cos x = -1$$




$$\cos\theta = \frac{adj}{hyp} = \frac{2}{1}$$

No Solution

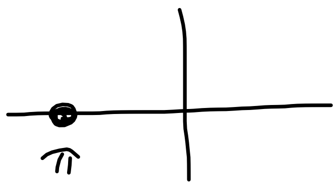
$$\cos x = \frac{ADJ}{HYP} = \frac{-1}{1}$$

left

on x-axis

$$x = \pi$$

General?




$$\theta = \pi + \underbrace{2\pi}_{\text{full cycle}} n, \; n \in I$$

full cycle

y=cos θ

3. Find the solutions for $\cos^2 x + 2\sin x - 2 = 0$, where $0 \le x < 2\pi$ Then write the general solution of the equation.

Issue?

cos x and sin x

"two variables, one equation"

Know: $\cos^2\theta + \sin^2\theta = 1$

$$(1 - \sin^2 x) + 2\sin x - 2 = 0$$

$$0 = \sin^2 x - 2\sin x + 1$$

$$A^2 - 2A + 1$$
$$= (A-1)(A-1)$$

$$0 = (\sin x - 1)(\sin x - 1)$$

$$\sin x = 1$$

$$\frac{opp}{hyp} = \frac{1}{1}$$ pos ∴ up

on y-axis, up

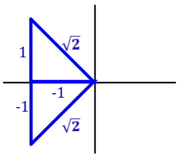
$$x = \frac{\pi}{2}$$

$$x = \frac{\pi}{2} + 2\pi n,\ n \in I$$

**Trig Equations**

4. Solve $\cos 3x = -\frac{\sqrt{2}}{2}$, where $0 \le x < 2\pi$

$\cos\theta = -\frac{\sqrt{2}}{2}$ $\quad \frac{\text{adj (neg)}}{\text{hyp}}$ $\quad \frac{-1}{\sqrt{2}}$ $\quad 45$




$\theta = 135^\circ = \frac{3\pi}{4}$

$\theta = 225^\circ = \frac{5\pi}{4}$

HORIZON STR $\frac{1}{3}$

$3x = \frac{3\pi}{4}$ $\qquad 3x = \frac{5\pi}{4}$

$x = \frac{\pi}{4}$ $\qquad x = \frac{5\pi}{12}$

$y = \cos 3x$

period $= \frac{2\pi}{3}$

OR $120^\circ$

$x = \frac{\pi}{4} + \frac{2\pi}{3} = \frac{11\pi}{12}$

$\frac{3\pi}{12} + \frac{8\pi}{12}$

$x = \frac{\pi}{4} + \frac{2\pi}{3} + \frac{2\pi}{3} = \frac{19\pi}{12}$

$x = \frac{5\pi}{12} + \frac{2\pi}{3} = \frac{13\pi}{12}$

$x = \frac{5\pi}{12} + \frac{8\pi}{12} + \frac{8\pi}{12} = \frac{21\pi}{12} = \frac{7\pi}{4}$

$x = \frac{\pi}{4}, \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}, \frac{7\pi}{4}$

# Trig Equations

5. If $\sec 2x + \frac{1}{\cos x} = 0$ where $0 \le x < \pi$, solve for $x$.

Issues: $\underbrace{\sec}\, 2x$, $\underbrace{\cos}\, x$ ratios

$\underbrace{2x, x}$ stretches

* $\boxed{\cos\theta = \frac{1}{\sec\theta}}$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos(A+B) = \cos A\cos B - \sin A \sin B$$

$$\cos(A+A) = \cos A \cos A - \sin A \sin A$$

together

$$\cos^2\theta + \sin^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

THEN

$$\cos 2\theta = \cos^2\theta - (1 - \cos^2\theta)$$

* $\boxed{\cos 2\theta = 2\cos^2\theta - 1}$

$$\sec 2x + \frac{1}{\cos x} = 0$$

$$\left[\frac{1}{\cos 2x} + \frac{1}{\cos x} = 0\right](\cos x)(\cos 2x)$$

$$\cos x + \cos 2x = 0$$

$$\cos x + \left(2\cos^2 x - 1\right) = 0$$

$$2\cos^2 x + \cos x - 1 = 0$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

$\cos x = \frac{1}{2}$ $\quad\quad$ $\cos x = -1$

$x = \frac{\pi}{3}$ $\quad\quad$ $x = \pi$

$0 \le x < \pi$

quad I, II

But domain

$\therefore x \ne \pi$

**Trig Equations**

6. Find the solutions for $\cos x = 1 + \sin x$ where $0 \le x < 2\pi$

$$\cos^2\theta + \sin^2\theta = 1 \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos\theta = \sqrt{1 - \sin^2\theta}$$

$$\cos x = \sin x + 1$$

$$\sqrt{1 - \sin^2 x} = \sin x + 1$$

ISOLATE $\sqrt{\phantom{x}}$

SQUARE

$(\sin x + 1)(\sin x + 1)$

$$1 - \sin^2 x = \sin^2 x + 2\sin x + 1$$

$$0 = 2\sin^2 x + 2\sin x$$

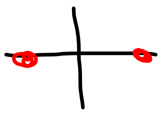
$$0 = 2\sin x(\sin x + 1)$$

$$\sin x = 0 \qquad \sin x = -1$$

$\frac{opp}{hyp} = \frac{0}{1}$

$$x = 0, \pi \qquad x = \frac{3\pi}{2}$$

$0 \le x < 2\pi$



EXT

| | $\cos x$ | $1 + \sin x$ |
|---|---|---|
| $x = \pi$ | $-1$ | $1 + 0$ <br> $1$ |

**Trig Equations**

**Homework:** Page 291 #1ad, 3ac, 4ce

**1.** Solve for $x$ in the interval $[0, 2\pi]$.

(a) $\sin x = \dfrac{\sqrt{3}}{2}$ (b) $\cos x = \frac{1}{2}$ (c) $\tan x = -1$

(d) $\sec x = -2$ (e) $\sin x = -\frac{1}{2}$ (f) $\cos^2 x = \frac{1}{4}$

**3.** Solve for $x$ in the given interval.

(a) $\sin x - \sin x \tan x = 0$, $[0, \pi]$

(b) $\sin x \tan 3x = 0$, $[-\pi, 0]$

(c) $6 \sin^2 x - 5 \cos x - 2 = 0$, $[0, 2\pi]$

**4.** Solve for $x$.

(a) $\cos 2x = \cos^2 x$, $-\pi \leqslant x \leqslant \pi$

(b) $\sin 2x = \cos x$, $-\pi \leqslant 2x \leqslant \pi$

(c) $\cos^2 x - 2 \sin x \cos x - \sin^2 x = 0$, $0 \leqslant 2x \leqslant \pi$

(d) $\tan 2x = 8 \cos^2 x - \cot x$, $0 \leqslant x \leqslant \dfrac{\pi}{2}$

(e) $\tan x + \sec 2x = 1$, $-\dfrac{\pi}{2} \leqslant x \leqslant \dfrac{\pi}{2}$

# Trig Equations

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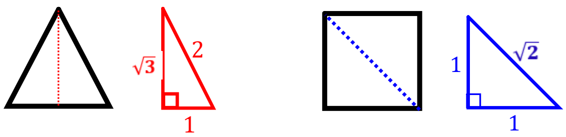


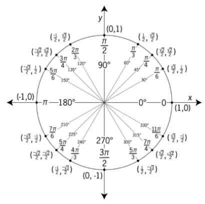

State the exact values of:

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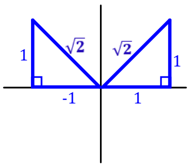
$\text{Sin } \frac{\pi}{6} = \frac{1}{2}$ $\quad$ $\text{Sin } \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\quad$ $\text{Sin } \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\text{Tan } \frac{\pi}{6} = \frac{1}{\sqrt{3}}$ $\quad$ $\text{Tan } \frac{\pi}{3} = \frac{\sqrt{3}}{1}$ $\quad$ $\text{Tan } \frac{\pi}{4} = \frac{1}{1}$

**Examples:**

1. Find the exact value(s) of $\theta$ where $0^0 \le \theta < 360^0$

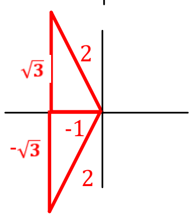



a) $\sin\theta = \frac{1}{\sqrt{2}}$ $\frac{\text{opp}}{\text{hyp}}$

reference angle = 45

$\boldsymbol{\theta = 45°, 135°}$

b) $\cos\theta = \frac{\sqrt{2}}{2}$

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$\boldsymbol{\theta = 120°, 240°}$

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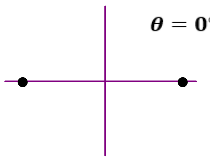

$\boldsymbol{\theta = 0°, 180°}$

f) $\cos\theta = -1$