

Exercise 4.3 The First Derivative Test

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1-4 every 2<sup>nd</sup> letter

- B 1. Find the local maximum and minimum values of  $f$ .
- (a)  $f(x) = 3x^2 - 4x + 13$       (b)  $f(x) = x^3 - 12x - 5$   
 (c)  $f(x) = 2 + 5x - x^5$       (d)  $f(x) = x^4 - x^3$
2. Find the critical numbers, intervals of increase and decrease, and local maximum values of the function. Then use this information to sketch the graph of  $f$ .
- (a)  $f(x) = 2 + 6x - 6x^2$   
(b)  $f(x) = x^3 - 9x^2 + 24x - 10$   
(c)  $g(x) = 1 + 3x^2 - 2x^3$   
(d)  $g(x) = 3x^4 - 16x^3 + 18x^2 + 1$   
(e)  $h(x) = x^4 - 8x^2 + 6$   
(f)  $h(x) = 3x^5 - 5x^3$
3. Find the local maximum and minimum values of  $f$ .
- (a)  $f(x) = 2x^{\frac{2}{3}}(3 - 4x^{\frac{1}{3}})$       (b)  $f(x) = \frac{x^2}{x^2 - 1}$   
 (c)  $f(x) = x\sqrt{4 - x}$       (d)  $f(x) = x\sqrt{1 - x^2}$
4. Find the absolute maximum or minimum value of the function.
- (a)  $f(x) = 27 + x - x^2$       (b)  $f(x) = 3 - \frac{1}{\sqrt{x^2 + 1}}$   
 (c)  $g(x) = \frac{x^2 - 1}{x^2 + 1}$       (d)  $g(x) = \frac{x^2 - x + 1}{x^2 + 1}, x \geq 0$

$$1c) \quad f(x) = 2 + 5x - x^5$$

$$f'(x) = 5 - 5x^4$$

$$\frac{dy}{dx} = 0$$

$$0 = 5 - 5x^4$$

$$5x^4 = 5 \quad \xrightarrow{\text{OR}} \quad x^4 = 1$$

$$x = \pm 1$$

$$0 = 5(1-x^4)$$

$$0 = (1-x^2)(1+x^2)$$

$$0 = (1-x)(1+x)(1+x^2)$$

$$x=1 \quad x=-1 \quad \text{so } 1+x^2=0 \\ \text{no solution}$$

	$5 - 5x^4$	$f'(x)$	$f(x)$
$-\infty, -1$	NEG		DEC
$-1, 1$	POS		INC
$1, \infty$	NEG		DEC

$\min f(-1)$   
 $\max f(1)$

$$f(-1) = 2 + 5(-1) - (-1)^5 = 2 + 5(-1) - (-1) = -2 \quad \text{minimum value}$$

$$f(1) = 2 + 5(1) - (1)^5 = 6 \quad \text{maximum value}$$

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3a)  $f(x) = 2x^{2/3}(3 - 4x^{-1/3})$

$$f(x) = 6x^{2/3} - 8x$$

$$f'(x) = 4x^{-1/3} - 8$$

$$\frac{dy}{dx} = 0$$

$$0 = 4x^{-1/3} - 8$$

$$8 = \frac{4}{x^{1/3}}$$

$$x^{1/3} = \frac{1}{2}$$

$$\boxed{x = \frac{1}{8}}$$

$$f'(x) = \frac{4}{x^{1/3}} - 8$$

$$0 = \frac{4}{x^{1/3}} - 8$$

$$8 = \frac{4}{x^{1/3}}$$

$$\frac{dy}{dx} = \text{undefined}$$

$$\boxed{x = 0}$$

	$\frac{4}{x^{1/3}} - 8 = f'(x)$	$f(x)$	
$(-\infty, 0)$	neg	dec	local min $f(0)$
$(0, \frac{1}{8})$	pos	inc	local max $f(\frac{1}{8})$
$(\frac{1}{8}, \infty)$	neg	dec	

$$f(x) = 2x^{2/3}(3 - 4x^{-1/3}) = 6x^{2/3} - 8x$$

$$f(0) = 0$$

$$f(\frac{1}{8}) = 6\left(\sqrt[3]{\frac{1}{8}}\right)^2 - 8\left(\frac{1}{8}\right)$$

$$= 6\left(\frac{1}{4}\right) - 1$$

$$= \frac{3}{2} - \frac{2}{2}$$

$$= \frac{1}{2}$$

domain  $x \leq 4$

3c)  $f(x) = x\sqrt{4-x}$

$$f = x$$

$$f' = 1$$

$$g = (4-x)^{1/2}$$

$$g' = \frac{1}{2}(4-x)^{-1/2}(-1)$$

$$f'g + fg'$$

$$f'(x) = (1)(4-x)^{1/2} + (x)(-\frac{1}{2})(4-x)^{-1/2}$$

$$f'(x) = \frac{1}{2}(4-x)^{-1/2} [2(4-x) - x]$$

$$f'(x) = \frac{8-3x}{2\sqrt{4-x}}$$

$$\frac{dy}{dx} = 0$$

$$8-3x=0$$

$$8=3x$$

$$x = \frac{8}{3}$$

$$\frac{dy}{dx} = \text{undefined}$$

$$\begin{cases} 4-x=0 \\ x=4 \end{cases}$$

	$8-3x$	$2\sqrt{4-x}$	$f'(x)$	$f(x)$
$(-\infty, \frac{8}{3})$	+	+	+	INC
$(\frac{8}{3}, 4)$	-	+	-	DEC
$(4, \infty)$	NOT IN DOMAIN, no function $\therefore$ no slope			

$$f(\frac{8}{3}) = \frac{8}{3}\sqrt{4-\frac{8}{3}}$$

$$\frac{12}{3} - \frac{8}{3} = \frac{4}{3}$$

$$= \frac{8}{3}\sqrt{\frac{4}{3}} \quad \text{OR} \quad \frac{8}{3}\left(\frac{2}{\sqrt{3}}\right)$$

$$= \frac{16}{3\sqrt{3}} \quad \text{OR} \quad \frac{16\sqrt{3}}{9}$$

4a  $f(x) = 27 + x - x^2$

$$f'(x) = 1 - 2x$$

$$\frac{dy}{dx} = 0$$

$$0 = 1 - 2x$$

$$2x = 1$$

$$x = \frac{1}{2}$$

$1 - 2x$	$f'(x)$	$f(x)$
$(-\infty, \frac{1}{2})$	pos	inc
$(\frac{1}{2}, \infty)$	neg	dec

$\max f(\frac{1}{2})$

$$f(\frac{1}{2}) = 27 + \frac{1}{2} - (\frac{1}{2})^2$$

$$= \frac{108}{4} + \frac{2}{4} - \frac{1}{4}$$

$= \frac{109}{4}$ , is absolute max of function

4c)  $g(x) = \frac{x^2 - 1}{x^2 + 1}$

$$f = x^2 - 1$$

$$f' = 2x$$

$$g = x^2 + 1$$

$$g' = 2x$$

$$\frac{f'g - fg'}{g^2}$$

$$g'(x) = \frac{2x(x^2 + 1) - (x^2 - 1)(2x)}{(x^2 + 1)^2}$$

$$= \frac{2x[x^2 + 1 - x^2 + 1]}{(x^2 + 1)^2} \quad \text{ooo [2]}$$

$$= \frac{4x}{(x^2 + 1)^2}$$

$$\frac{dy}{dx} = 0 \quad 4x = 0$$

$$x = 0$$

$$\frac{dy}{dx} = \text{undefined}$$

$$(x^2 + 1)^2 = 0$$

no solution  $\therefore$  never

divide by zero

	$4x$	$(x^2 + 1)^2$	$f'(x)$	$f(x)$
$-\infty, 0$	NEG	POS	NEG	DEC
$0, \infty$	POS	POS	POS	INC

$\Rightarrow \min f(0)$

$$g(0) = \frac{(0)^2 - 1}{(0)^2 + 1}$$

$$g(0) = -1$$

Absolute minimum value is  $-1$ .