## 1.0 Anti Der

# *Antiderivatives*

**Outcomes:** Find the antiderivatives of functions.

**Part A – Polynomial Functions:**

**Review:** Find the derivative of

a) $y = x^2$ b) $y = x^2 + 3$ c) $y = x^2 - 7$

$y' = 2x$ $y' = 2x$ $y' = 2x$

$f'(x) = 2x$

$f(x) = x^2 + c$

$c$ = constant

**Investigate:** If $f'(x) = 3x^2$, what is $f(x)$.

$f(x) = \frac{3x^3}{3} + c \quad \therefore f(x) = x^3 + c$

- **The original function from which the derivative was obtained is termed the antiderivative, the integral or the primitive.**
- **Anti-differentiation is the inverse operation of differentiation.**
- **Like the derivative the antiderivative may be represented in various ways.**

If $y = f'(x)$ is the derived function then $y = F(x)$ is the antiderivative.

Remember that any function in the form $y = F(x) + C$ will have the same derivative.

$$\frac{d}{dx}(F(x) + C) = f(x)$$

The antiderivative, or integral, is commonly represented by the following:

$\int f(x)dx$ 'read - the integral of $f(x)$ with respect to x'

$\int 3x^2 = x^3 + C$

Sum = $\int$

So doing an antiderivative is like doing a derivative backward.

- Derivatives, we reduce by a degree and multiply.
- Antiderivatives we

**1.** Find the general antiderivative of $f(x) = 1$ *or* $\boxed{f = x^0}$

$F(x) = \frac{x^1}{1} + c$

$F(x) = x + c$

**2.** Find the indefinite integral when $F(x) = \int x dx$

$F(x) = \frac{x^2}{2} + c = \frac{1}{2}x^2 + c$

**3.** Find the general antiderivative of $f(x) = 2x + 3x^0$

$F(x) = \frac{2x^2}{2} + \frac{3x^1}{1} + c$

$F(x) = x^2 + 3x + c$

**4.** Find the (most general) antiderivative of $f(x) = 4x^3 - 6x^2 + 11$

$F(x) = \frac{4x^4}{4} + -\frac{6x^3}{3} + \frac{11x^1}{1} + C$ $\quad F(x) = x^4 - 2x^3 + 11x + C$

**5.** Integrate $F(x) = \int x^2 dx$ $\quad F(x) = \frac{1}{3}x^3 + C$

function x was reduced

increase function x ...

**6.** Find the general primitive of $f(x) = x^3$

$F(x) = \frac{1}{4}x^4 + C$

**7.** Find the antiderivative of $f(x) = \sqrt{x}$

$f(x) = x^{1/2}$

$F(x) = \frac{x^{3/2}}{3/2} + C$

$F(x) = \frac{2}{3}x^{3/2} + C$

**Part B – TRIGONOMETRY:** start with?

**1.** Find the antiderivative of each of the following

a) $f(x) = sinx$

$F(x) = -\cos x + C$

b) $f(x) = sin(3x)$

$F(x) = \frac{-\cos(3x)}{\frac{d}{dx}(3x)} + C = -\frac{1}{3}\cos(3x) + C$

c) $f(x) = cosx$

$F(x) = \sin x + C$

d) $f(x) = cos(5x)$

$F(x) = \frac{\sin(5x)}{5} + C$

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2. Find the antiderivative of on the interval

$f(x) = \cos x - \sin x$

$F(x) = \sin x + \cos x + c$

3. Find the most general antiderivative of $f(x) = \underline{\sin x \cos x}$

$\frac{1}{2}(2 \sin x \cos x)$

$f(x) = \frac{1}{2} \sin(2x)$

$F(x) = \frac{1}{2}\left[-\frac{\cos(2x)}{2}\right] + c$

$F(x) = -\frac{1}{4}\cos(2x) + c$

**Part C – EXPONENTS AND LOGARITHM:**

1. Find the antiderivative of *f(x)*, $f(x) = -3e^{-x} + 6e^{2x}$

$F(x) = \frac{-3e^{-x}}{\frac{d}{dx}(-x)} + \frac{6e^{2x}}{\frac{d}{dx}(2x)} + c$

$F(x) = 3e^{-x} + 3e^{2x} + c$

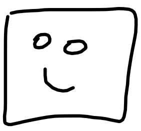
2. Find the antiderivative of *f(x)* on the interval $(0, \infty)$

$f(x) = \frac{2}{x^2} - \frac{5}{x} + x$

$f(x) = 2x^{-2} - 5x^{-1} + x$ ooo polynomial

"add degree divide..."

$F(x) = \frac{2x^{-1}}{-1} - \frac{5x^0}{0} + \frac{x^2}{2} + c$



ooo

$[y = \ln x] \Rightarrow \frac{dy}{dx} = \frac{1}{x}$

$F(x) = \frac{2x^{-1}}{-1} - 5\ln x + \frac{x^2}{2} + c$

$F(x) = -\frac{2}{x} - 5\ln x + \frac{1}{2}x^2 + c$

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**Complete the following table**

| **Function:** $f(x) =$ | | **Particular most general antiderivative** |
|---|---|---|
| 0 | $\frac{dy}{dx} = 0$ | $F(x) = c$ |
| 1 | $m = 1$ | $F(x) = x + c$ |
| $x^n$ | $\frac{dy}{dx} = x^n$ | $F(x) = \frac{x^{n+1}}{n+1} + c$ |
| $\frac{1}{x}$ | $y' = \frac{1}{x}$ | $F(x) = \ln\lvert x\rvert + c$ |
| $e^{kx}$ | $f'(x) = e^{kx}$ | $F(x) = \frac{e^{kx}}{k} + c$ |
| $\cos kx$ | $\frac{dy}{dx} = \cos(kx)$ | $F(x) = \frac{\sin(kx)}{k} + c$ |
| $\sin kx$ | $y' = \sin kx$ | $y = \frac{-\cos(kx)}{k} + c$ |

**Homework:** Pg 408 #1 2a, 2b, 3b, 3c, 3d, 4, 6a, 8bc