

1.0 Factoring and Radicals 2020

Math 31

Factoring and Radical Skills – use past knowledge to factor and rationalize expressions.

Multiply: $(2x + 3)(x + 2)(x - 1)$

... know first and last term, use this to help with factoring.

1. Factor

- Factor theorem; synthetic division

a) $2x^3 + x^2 - 5x + 2$

b) $x^3 - x^2 - 14x + 24$

c) $x^3 - 8$

d) $x^3 + 8$

e) $x^3 - 27$

- Pattern in math: “sum and/or difference of cubes”

f) $x^3 - 125$

g) $27x^3 + 125y^3$

- Common Factoring: factor largest number, smallest x

h) $12x^3 + 18x$

i) $3x^3 + 7x^2$

j) $5x^2 + 3x + x^{-1}$

k) $3(x + 4)^{\frac{1}{2}} - 5(x + 4)^{-\frac{1}{2}}$

2. Simplify radical expressions

- Radicals – rationalize denominators

a) $\frac{\sqrt{3}}{\sqrt{5}}$

b) $\frac{\sqrt{3}}{\sqrt{5} + \sqrt{2}}$

- Radicals – rationalize numerators

c) $\frac{\sqrt{x+2}}{\sqrt{x}}$

d) $\frac{\sqrt{x+9}-3}{x}$

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Multiply: $(2x+3)(x+2)(x-1)$

... know first and last term, use this to help with factoring.

$$\begin{aligned} & (2x+3)(x^2+x-2) \\ &= 2x^3 + 2x^2 - 4x + 3x^2 + 3x - 6 \\ &= 2x^3 + 5x^2 - x - 6 \end{aligned}$$

a) $2x^3 + x^2 - 5x + 2 \dots f(1) = 0$

b) $x^3 - x^2 - 14x + 24$

$$\begin{array}{r|rrrr} 1 & 2 & 1 & -5 & 2 \\ & & 2 & 3 & -2 \\ \hline & 2 & 3 & -2 & 0 \end{array}$$

$$= (2x^2 + 3x - 2)(x - 1)$$

$$= (2x - 1)(x + 2)(x - 1)$$

a) $2x^3 + x^2 - 5x + 2$

b) $x^3 - x^2 - 14x + 24$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & -14 & 24 \\ & & 2 & 2 & -24 \\ \hline & 1 & 1 & -12 & 0 \end{array}$$

$$(x^2 + x - 12)(x - 2)$$

$$= (x + 4)(x - 3)(x - 2)$$

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c) $x^3 - 8 \dots f(2) = 0$

d) $x^3 + 8$

e) $x^3 - 27$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 0 & -8 \\ & & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & 0 \end{array}$$

$$x^3 - 8 = (x - 2)(x^2 + 2x + 4)$$

c) $x^3 - 8$

d) $x^3 + 8 \dots f(-2) = 0$

e) $x^3 - 27$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 0 & 8 \\ & & -2 & 4 & -8 \\ \hline & 1 & -2 & 4 & 0 \end{array}$$

$$x^3 + 8 = (x + 2)(x^2 - 2x + 4)$$

c) $x^3 - 8$

d) $x^3 + 8$

e) $x^3 - 27 \dots f(3) = 0$

$$\begin{array}{r|rrrr} 3 & 1 & 0 & 0 & -27 \\ & & 3 & 9 & 27 \\ \hline & 1 & 3 & 9 & 0 \end{array}$$

$$x^3 - 27 = (x - 3)(x^2 + 3x + 9)$$

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- Pattern in math: "sum and/or difference of cubes"

f) $x^3 - 125$

g) $27x^3 + 125y^3$

$$f) \quad x^3 - 125 = (x - 5)(x^2 + 5x + 25)$$

$$g) \quad 27x^3 + 125y^3 = \\ = (3x + 5y)(9x^2 - 15xy + 25y^2)$$

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- Common Factoring: factor largest number, smallest x

h) $12x^3 + 18x$

i) $3x^3 + 7x^2$... x^2

j) $5x^2 + 3x + x^{-1}$... x^{-1}

k) $3(x+4)^{\frac{1}{2}} - 5(x+4)^{-\frac{1}{2}}$

h) $6x(2x^2 + 3)$

i) $x^2(3x + 7)$

j) $x^{-1}(5x^3 + 3x^2 + 1)$

k) $(x+4)^{-1/2} [3(x+4) - 5(1)]$

$(x+4)^{-1/2} (3x+7)$

OR $\frac{3x+7}{\sqrt{x+4}}$

2. Simplify radical expressions

- Radicals – rationalize denominators

a) $\frac{\sqrt{3}}{\sqrt{5}} \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{15}}{5}$

b) $\frac{\sqrt{3}}{\sqrt{5}+\sqrt{2}} \left(\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} \right) = \frac{\sqrt{15} \cdot \sqrt{6}}{5-2} = \frac{\sqrt{15} - \sqrt{6}}{3}$

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- Radicals – rationalize numerators

$$\text{c) } \frac{\sqrt{x+2}}{\sqrt{x}} \left(\frac{\sqrt{x-2}}{\sqrt{x-2}} \right) = \frac{x-4}{\sqrt{x}(\sqrt{x-2})}$$

$$\text{d) } \frac{\sqrt{x+9}-3}{x}$$

$$\left(\frac{\sqrt{x+9}-3}{x} \right) \left(\frac{\sqrt{x+9}+3}{\sqrt{x+9}+3} \right)$$

$$= \frac{(x+9) - 9}{(x)(\sqrt{x+9}+3)}$$

$$= \frac{x}{(x)(\sqrt{x+9}+3)}$$

$$= \frac{1}{\sqrt{x+9}+3}$$