Lesson 1

Maximum and Minimum Number Problems

Objectives: Solve maximum and minimum problems using derivatives

Warm Up: How do we find the maximum and/or minimum values of any curve.

Investigate: Different pairs of numbers may have the same sum but they will have different products. Among these products it is possible to find a maximum value. Trial and error could be used to identify these values, but that procedure would become time-consuming as the given sum increases. The following examples shows a way that calculus solves this and similar problems.

1. Find two positive numbers whose sum is 15 and whose product is a maximum.

I

$x+y=15$

$y=15-x$

II

$f(x)=xy$

$f(x)=(x)(15-x)$

$f(x)=15x-x^2$

$\frac{dy}{dx}=0$

$f'(x)=15-2x$

$0=15-2x$

$x=7.5$

$y=7.5$

1.0 Maxmin numbers

2. Find two numbers whose difference is 100 and whose product is a minimum.

$x - y = 100$

$y = x - 100$

$f(x) = xy$

$f(x) = (x)(x - 100)$

$f(x) = x^2 - 100x$

$f'(x) = 2x - 100$

$0 = 2x - 100$

$x = 50$

$y = x - 100$

$y = -50$

1.0 Maxmin numbers

3. Find two positive numbers whose product is 108 and the sum of the first number plus three times the second number is a minimum.

$xy = 108$

$y = \frac{108}{x}$

$f(x) = x + 3y$

$f(x) = x + 3\left(\frac{108}{x}\right)$

$f(x) = x + 324x^{-1}$

$f'(x) = 1 - 324x^{-2}$

$0 = 1 - \frac{324}{x^2}$

$\frac{324}{x^2} = 1$

$324 = x^2$

$x = 18$

$y = \frac{108}{x}$

$y = 6$

1.0 Maxmin numbers

4. Find two positive numbers whose sum is 30 and the square of one number plus twice the square of the other is
   a) a minimum
   b) a maximum

a) $x+y=30$ $\quad f(x)=x^2+2y^2$

$y=30-x$ $\quad f(x)=x^2+2(30-x)^2$

$\frac{dy}{dx}=0$

$f'(x)=2x+4(30-x)^1(-1)$

$0=2x-4(30-x)$

$0=2x-120+4x$

$120=6x$

$y=10$ $\quad x=20$

b) $0<x\leq 30$

$x=0$
$y=30$

$x=30$
$y=0$

$f(x)=x^2+2y^2$
max

1.0 Maxmin numbers

Homework:

1. The sum of two natural numbers is 28. Find these two numbers if their product is a maximum.

2. Find two consecutive natural numbers if the sum of the larger number and four times the reciprocal of the smaller number is a minimum.

3. The sum of two positive numbers is 4. If the sum of their cubes is a minimum, what are these numbers?

4. Find two numbers that satisfy the conditions that the sum of the first number and twice the second number is 100, and the product is a maximum.

Solutions:

1. The two natural numbers are 14 and 14.
2. The two numbers are 2 and 2 + 1 = 3.
3. The two numbers are 2 and 2.
4. The two numbers are 50 and 25.