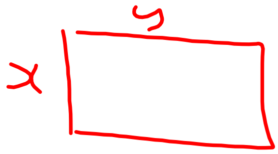
Lesson 2

Maximum and Minimum Geometric Problems

Objectives:

Solve maximum and minimum problems using derivatives

1. Find the dimensions of a rectangle if the perimeter is 28 cm and the area is a maximum. What is the maximum area?



know $2x+2y=28$

$x+y=14$

$y=14-x$

$f(x)=$ area

$f(x)=(x)(y)$

$f(x)=(x)(14-x)$

$f(x)=14x-x^2$

slope = zero
MAX
$f'(x)=0$

$f'(x)=14-2x$

$0=14-2x$

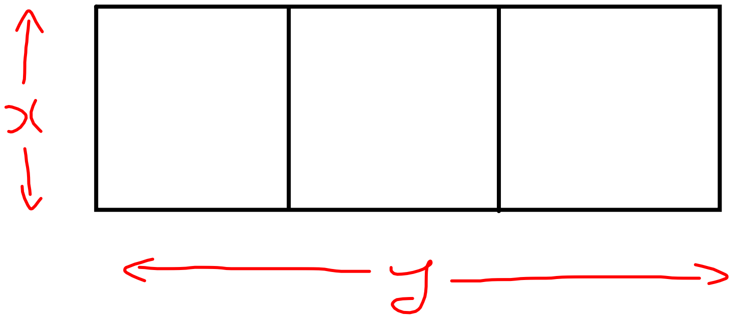
$x=7$

$y=14-x$

$y=7$

MAX AREA $=(7)(7)=49\text{ cm}^2$

2. A farmer wants to fence a rectangular enclosure for his horses and then divide it into thirds with fences parallel to one side of the rectangle. If he has 2000m of fencing, find the area of the largest rectangle that can be enclosed.



$4x + 2y = 2000$

$f(x) = xy$

$2x + y = 1000$

$y = 1000 - 2x$

$f(x) = (x)(1000 - 2x)$

$f(x) = 1000x - 2x^2$

$f'(x) = 1000 - 4x$

$0 = 1000 - 4x$

$y = 1000 - 2x$

$x = 250$

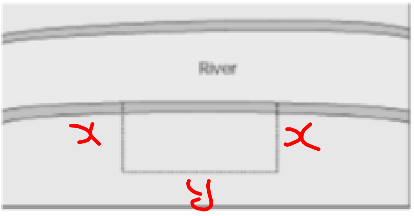
$y = 500$

$A = (250)(500)$

$A = 125000\ m^2$

12.5 ha

3. A farmer wishes to fence part of a rectangular field along a straight river as shown in the following diagram. It is not necessary to fence the side bordering the river. The area of the rectangular field is to be $\boxed{1800 \text{ m}^2}$ and the farmer wishes to use the least length of fencing material. What should the dimensions of the rectangular field be?



$f(x) = 2x + y$

$xy = 1800$

$y = \frac{1800}{x}$

$f(x) = 2x + 1800x^{-1}$

$f'(x) = 2 - 1800x^{-2}$

$0 = 2 - 1800x^{-2}$

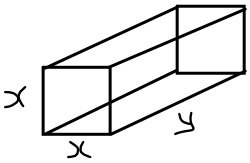
$\frac{1800}{x^2} = 2$

$900 = x^2$

$x = 30$

$y = \frac{1800}{30} = 60$

4. A rectangular box with two squares ends has a total surface area 150 cm². Find the dimensions of the box if the volume is a maximum. What is the maximum volume?



$f(x) = x^2 y$

$2x^2 + 4xy = 150$

$x^2 + 2xy = 75$

$2xy = 75 - x^2$

$y = \frac{75 - x^2}{2x}$

$f(x) = (x^2)\left(\frac{75 - x^2}{2x}\right)$

$f(x) = \frac{1}{2}x(75 - x^2)$

$f(x) = \frac{75}{2}x - \frac{1}{2}x^3$

$\frac{dy}{dx} = 0$

$f'(x) = \frac{75}{2} - \frac{3}{2}x^2$

$0 = \frac{75}{2} - \frac{3}{2}x^2$

$0 = 75 - 3x^2$

$x^2 = 25$

$x = 5$

$y = \frac{75 - x^2}{2x}$

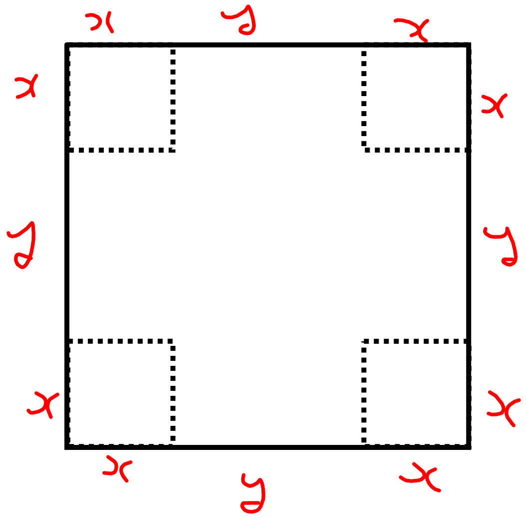
$y = \frac{75 - 25}{2(5)} = 5$

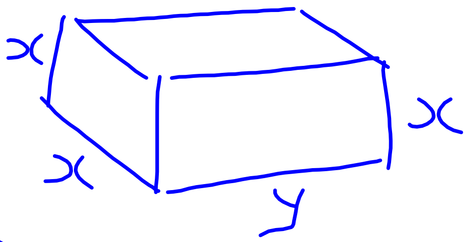
$V = 5 \times 5 \times 5$

$= 125 \text{ cm}^3$

5. An open box is to be made from a square piece of material $\boxed{\text{30 cm}}$ on a side, by cutting equal squares from each corner and turning up the sides. Find the volume of the largest box that can be made this way.

$$2x + y = 30$$





$$f(x) = (x)(x)(y)$$

$$f(x) = (x)(x)(30-2x)$$

$$f(x) = 30x^2 - 2x^3$$

$\frac{dy}{dx} = 0$

$$f'(x) = 60x - 6x^2$$

$$0 = 60x - 6x^2$$

$$0 = 6x(10-x)$$

$$x = 0 \qquad x = 10$$

$$y = 30 - 2x$$

$$y = 10$$

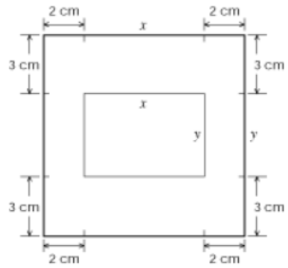
$$V = (10)(10)(10) = 1000 \text{ cm}^3$$

AREA

6. A rectangular page is to contain $\boxed{150\ \text{cm}^2}$ of printing. The margins at the top and bottom of the page are 3 cm. The margins at each side are 2 cm. What should the dimensions of the page be if the minimum amount of paper is used?

$xy = 150$

$y = \frac{150}{x}$



$f(x) = (x+4)(y+6)$

$f(x) = (x+4)\left(\frac{150}{x} + 6\right)$

$f(x) = 150 + \frac{600}{x} + 6x + 24$

$f(x) = 600x^{-1} + 6x + 174$

$f'(x) = -600x^{-2} + 6$

$0 = -600x^{-2} + 6$

$\frac{600}{x^2} = 6$

$x^2 = 100$

$x = 10$

$y = \frac{150}{x} = 15$

Paper $\begin{cases} 10+4 = 14 \\ 15+6 = 21 \end{cases}$

14 by 21 cm