

Lesson 3

More Maximum and Minimum Geometric Problems

Objectives: Solve Geometric problems

Skills: Find the distance between two points. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

- Find the distance between:
 - P₁ (3,2) and P₂ (-2,10)
 - P₁ (-1,-3) and P₂ (2,8)
- Do you need the square root to determine the max or min distance?

$$d = \sqrt{(-2 - 3)^2 + (10 - 2)^2}$$

$$d = \sqrt{25 + 64}$$

$$d = \sqrt{89}$$

$$d = \sqrt{(2 + 1)^2 + (8 + 3)^2}$$

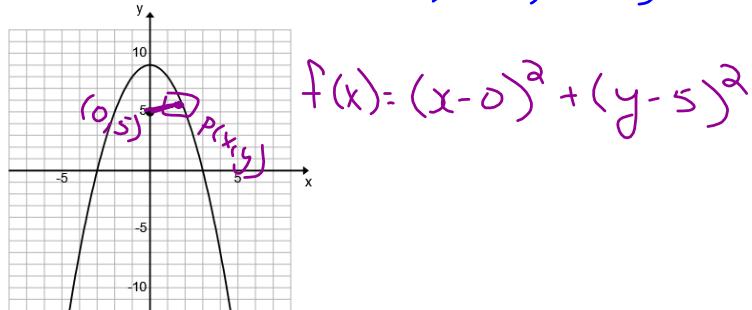
$$d = \sqrt{9 + 121}$$

$$d = \sqrt{130}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

1. Find the points on the graph of $y = 9 - x^2$ that are closest to the point $(0, 5)$.

$$f(x) = (x_2 - x_1)^2 + (y_2 - y_1)^2$$



$$f(x) = (x - 0)^2 + (9 - x^2 - 5)^2$$

$$f(x) = x^2 + (4 - x^2)^2$$

$$f'(x) = 2x + 2(4 - x^2) \cdot (-2x)$$

$$0 = 2x - 4x(4 - x^2)$$

$$0 = 2x - 16x + 4x^3$$

$$0 = 4x^3 - 14x$$

$$0 = 2x(2x^2 - 7)$$

$$2x = 0$$

$$2x^2 - 7 = 0$$

$$\boxed{x = 0}$$

$$x^2 = \frac{7}{2}$$

$$y = 9 - x^2$$

$$x = \pm \sqrt{\frac{7}{2}}$$

$$P(0, 9)$$

$$y = 9 - x^2$$

$$y = 9 - \frac{7}{2} \dots \frac{18}{2} - \frac{7}{2}$$

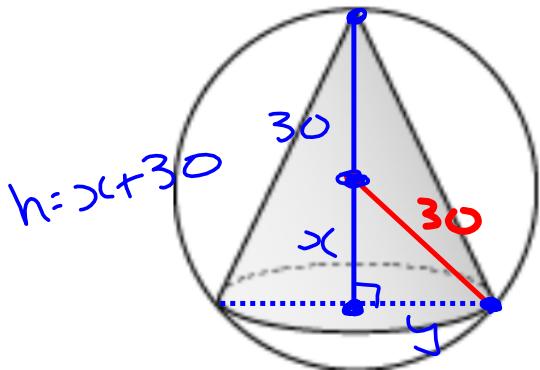
$$y = \frac{11}{2}$$

$$\boxed{P\left(\pm \sqrt{\frac{7}{2}}, \frac{11}{2}\right)}$$

two minimum points.

$$V = \frac{1}{3} \pi r^2 h$$

2. Find the volume of the largest right circular cone that can be inscribed in a sphere of radius 30 cm.



$$f(x) = \frac{1}{3} \pi r^2 h$$

$$f(x) = \frac{1}{3} \pi (900 - x^2)(x + 30)$$

$$\begin{aligned}x^2 + y^2 &= 30^2 \\y^2 &= 900 - x^2\end{aligned}$$

$$f(x) = \frac{1}{3} \pi [-x^3 - 30x^2 + 900x + 27000]$$

$$f'(x) = -3x^2 - 60x + 900$$

$$0 = -3(x^2 + 20x - 300)$$

$$0 = (x + 30)(x - 10)$$

$$x = -30 \quad \boxed{x = 10}$$

$$\text{height} = x + 30$$

$$h = 40$$

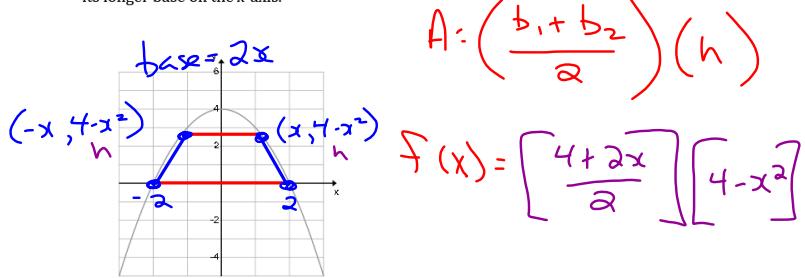
$$r^2 = 900 - x^2$$

$$r^2 = 800$$

$$V = \frac{1}{3} \pi (800)(40)$$

$$= \frac{32000\pi}{3}$$

3. Find the maximum area of a trapezoid inscribed in the function $y = 4 - x^2$ if the trapezoid has its longer base on the x-axis.



$$y = 4 - x^2$$

$$0 = 4 - x^2$$

$$x^2 = 4$$

$$x = \pm 2$$

$$\text{base} = 4$$

$$A = \left(\frac{b_1 + b_2}{2} \right) (h)$$

$$f(x) = \left[\frac{4+2x}{2} \right] \left[4-x^2 \right]$$

$$f(x) = (2+x)(4-x^2)$$

$$f(x) = -x^3 - 2x^2 + 4x + 8$$

$$f'(x) = -3x^2 - 4x + 4$$

$$0 = -3x^2 - 4x + 4$$

$$0 = 3x^2 + 4x - 4$$

$$0 = 3x^2 + 6x - 2x - 4$$

$$0 = (3x - 2)(x + 2)$$

$$x = \frac{2}{3} \quad x = -2$$

$$b_1 = 2 \left(\frac{2}{3} \right) = \frac{4}{3} \quad h = 0$$

$$h = 4 - x^2 = 4 - \left(\frac{2}{3} \right)^2$$

$$= \frac{36}{9} - \frac{4}{9}$$

$$= \frac{32}{9}$$

$$A = \left(\frac{4 + \frac{4}{3}}{2} \right) \left(\frac{16}{9} \right)$$

$$\left(\frac{12}{3} + \frac{4}{3} \right) \left(\frac{16}{9} \right)$$

$$\left(\frac{16}{3} \right) \left(\frac{16}{9} \right)$$

$$A = \frac{256}{27}$$

Homework:

1. Find the point on the parabola $y = 6 - x^2$ that is closest to the point $(0, 3)$
2. If an isosceles triangle is inscribed in a circle of radius 4 cm, find the dimensions of the isosceles triangle of maximum area.
3. Find the maximum volume of the largest right circular cone that can be inscribed in a sphere of radius 12 cm.
4. Find the dimensions of the right circular cylinder of largest volume that can be inscribed in a sphere of radius 30 cm.
5. Find the dimensions of a rectangle of maximum area inscribed in a circle of radius 6 cm.

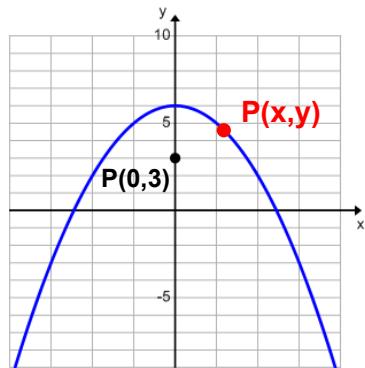
Solutions

1. $\left(\sqrt{2\frac{1}{2}}, 3\frac{1}{2}\right), \left(-\sqrt{2\frac{1}{2}}, 3\frac{1}{2}\right)$
2. The height of the triangle is 6 cm and the base is $4\sqrt{3}$ cm.
3. The maximum volume is $\frac{2048}{3}\pi \text{ cm}^3$
4. The radius of the right circular cylinder is $10\sqrt{6}$ cm and the height is $20\sqrt{3}$ cm.
5. The dimensions are $6\sqrt{2}$ cm by $6\sqrt{2}$ cm.

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HW – Applications of Derivatives.

1. Find the point on the parabola $y = 6 - x^2$ that is closest to the point $(0, 3)$



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(x - 0)^2 + (y - 3)^2}$$

$$y = 6 - x^2$$

$$d = \sqrt{(x - 0)^2 + (6 - x^2 - 3)^2}$$

$$d(x) = x^2 + (3 - x^2)^2$$

$$d'(x) = 2x + 2(3 - x^2)' (-2x)$$

$$d'(x) = 2x - 4x(3 - x^2)$$

$$0 = 2x - 12x + 4x^3$$

$$0 = 4x^3 - 10x$$

$$0 = 2x(2x^2 - 5)$$

$$2x = 0$$

$$2x^2 - 5 = 0$$

$$x = 0$$

$$x^2 = \frac{5}{2}$$

$$x = \pm \sqrt{\frac{5}{2}} = \pm \frac{\sqrt{10}}{2}$$

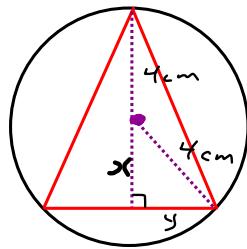
$$y = 6 - x^2$$

$$y = 6 - \frac{5}{2} \quad \frac{12}{2} - \frac{5}{2}$$

$$y = \frac{7}{2}$$

Points : $P\left(\pm \frac{\sqrt{10}}{2}, \frac{7}{2}\right)$

2. If an isosceles triangle is inscribed in a circle of radius 4 cm, find the dimensions of the isosceles triangle of maximum area.



$$A = \frac{bh}{2}$$

$$\text{height} = x + 4$$

$$x^2 + y^2 = 4^2$$

$$y = \sqrt{16 - x^2}$$

$$\text{base} = 2\sqrt{16 - x^2}$$

$$A(x) = \frac{(2\sqrt{16 - x^2})(x + 4)}{2}$$

$$A(x) = (16 - x^2)^{1/2} (x + 4)$$

$$f = (16 - x^2)^{1/2}$$

$$f' = \frac{1}{2}(16 - x^2)^{-1/2}(-2x)$$

$$f' = -x(16 - x^2)^{-1/2}$$

$$g = x + 4$$

$$g' = 1$$

$$A'(x) = -x(16 - x^2)^{-1/2}(x + 4) + (16 - x^2)^{1/2}(1)$$

$$A'(x) = (16 - x^2)^{-1/2} \left[-x(x + 4) + (16 - x^2) \right]$$

$$0 = \frac{-2x^2 - 4x + 16}{(16 - x^2)^{1/2}}$$

← cross multiply

$$0 = -2(x^2 + 2x - 8)$$

$$0 = -2(x + 4)(x - 2)$$

$$x = -4$$

$$x = 2$$

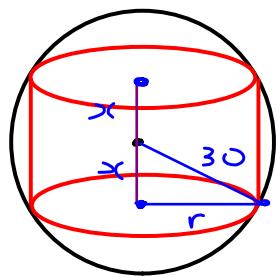
height = 0
min AREA

height = 6
max AREA

$$\text{base} = 2(\sqrt{16 - (2)^2})$$

$$\begin{aligned} &= 2\sqrt{12} \dots \sqrt{4\sqrt{3}} \\ &= 4\sqrt{3} \quad \text{2}\sqrt{3} \end{aligned}$$

4. Find the dimensions of the right circular cylinder of largest volume that can be inscribed in a sphere of radius 30 cm.



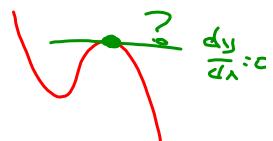
cylinder

$$V = \pi r^2 h$$

$$\text{height: } x + x = 2x \quad x^2 + r^2 = 30^2$$

$$r^2 = 900 - x^2$$

$$V(x) = \pi (900 - x^2)(2x)$$



$$V(x) = \pi [1800x - 2x^3]$$

$$V'(x) = \pi [1800 - 6x^2]$$

$$0 = \pi (1800 - 6x^2)$$

$$0 = 1800 - 6x^2$$

$$6x^2 = 1800$$

$$x^2 = 300 \quad \sqrt{100} \sqrt{3}$$

$$x = 10\sqrt{3}$$

Dimensions

$$h = 2(10\sqrt{3})$$

Radius

$$h = 20\sqrt{3}$$

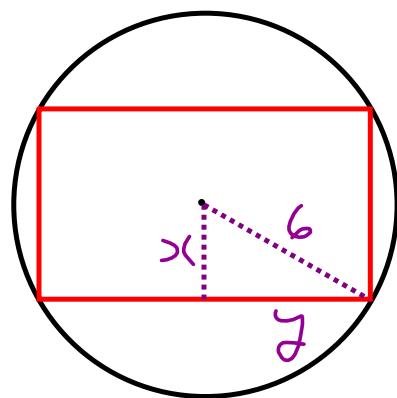
$$r^2 = 900 - x^2$$

$$r^2 = 900 - 300$$

$$r^2 = 600$$

$$r = 10\sqrt{6}$$

5. Find the dimensions of a rectangle of maximum area inscribed in a circle of radius 6 cm.



$$A = lw$$

$$l = 2x \quad w = 2y$$

$$x^2 + y^2 = 6^2$$

$$y = \sqrt{36 - x^2}$$

$$A(x) = (2x)(2\sqrt{36 - x^2})$$

$$A(x) = 4x(36 - x^2)^{1/2}$$