$1 + 0.50(1) = 1.5$ $\quad$ $1.5 + 1.50(0.50) = 2.25$

Derivative of the Exponential Function With Base e

Objectives: Find the derivative of exponential functions.

Warm up: Estimations of *e*

If you began walking at 1 km/h and then doubled your speed over a one-minute interval, you would be walking at 2 km/h. But suppose you increased your speed by 50% every half-minute. How fast would you be walking at the end of one minute?

Suppose you increased your speed by 25% every quarter-minute. What would your speed be at the end of one minute? Remember, your speed would be 1.25 times as fast every quarter-minute. Complete the chart below

Time Elapsed (s)	0	15	30	45	60
Speed (km/h)	1.00	1.25	1.5625	1.95	2.4414...

$1.00 + 1.00(0.25)$ $\quad$ $1.25 + 1.25(0.25)$

Generate an expression to find your speed at the end of one minute.

$1(1+0.25)^3 = 1.953125$ $\quad$ $(1.25)^4$

Suppose you increased your speed by $\frac{1}{10}$ for every tenth of a minute. What would your speed be at the end of one minute?

$(1+0.10)^{10} = (1.10)^{10} = 2.5937...$

Complete the table for each increase in speed for and equal portion of a minute.

Increase in speed	Speed at the end of 1 minute
$\frac{1}{10}$ $\left(1+\frac{1}{10}\right)^{10}$	2.5937424 6
$\frac{1}{1000}$ $\left(1+\frac{1}{1000}\right)^{1000}$	2.7169239...
$\frac{1}{100000}$ $\left(1+\frac{1}{100\,000}\right)^{100\,000}$	2.718268237...
$\frac{1}{10000000}$	2.71828169...
$\frac{1}{1000000000}$	2.718281827...

ten million

one billion

e
$n \to 1000000000$ $= 2.718281828\ldots$

Derivatives of $y = e^x$

$(5, 148.4..)$

e can be defined as: $e = \lim_{n\to\infty}\left[1+\frac{1}{n}\right]^n$

$y = (2.71828\ldots)^x$

Investigate:
Why is *e* such a special number?

Use your calculator to sketch $y = e^x$.
Find the values of e^x at $x = 1,3,5$

Find the derivative of $y = e^x$ at $x = 1,3,5$ using your calculator.

$f'(1)$

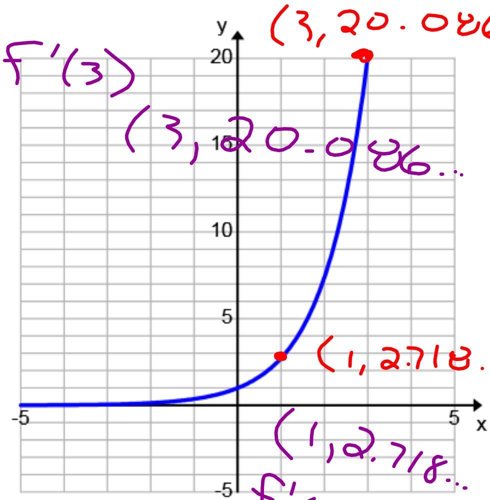
State the value of $\frac{dy}{dx}e^x$

$f(x) = f'(x)$

$f'(3)$

$(3, 20.086..)$

$(3, 20.086\ldots)$



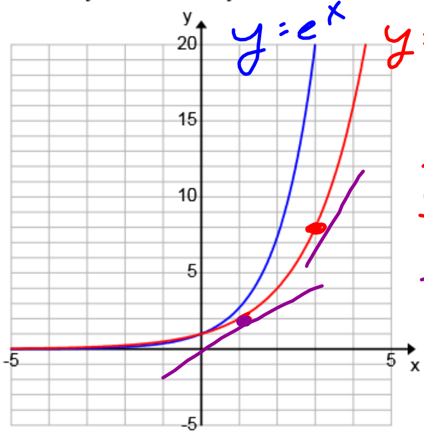
$(1, 2.718\ldots)$

$(1, 2.718\ldots)$

$f'(x)$

$y = 2^x$ and $y = e^x$

$y = e^x$ $y = 2^x$



$x = 1$
$f(1) = 2$
$f'(1) = 1.36$

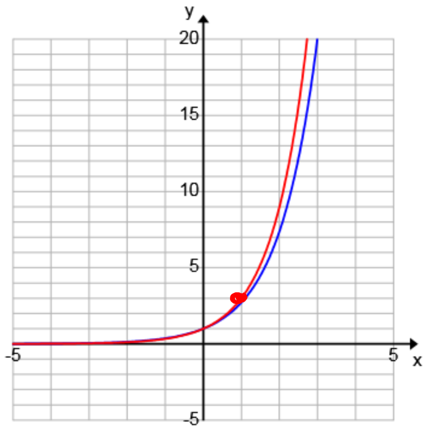
$x = 3$
$y = 2^x = 2^3$
$y = 8$

$f'(3)$
$= 5.54$

$y = 3^x$

$y = 3^x$ and $y = e^x$

$y = 2.7^x$



$f(1) = 3^1 = 3$
$f'(1) = 3.095\ldots$

$f(3) = 3^3 = 27$
$f'(3) = 29.66$

2.0 Derive Base e.2018

Chain Rule: $f(x) = e^u$ then $f'(x) = e^u \cdot \frac{du}{dx}$

1. Differentiate

a) $y = x^3 e^x$

PRODUCT

$f = x^3 \quad g = e^x$

$f' = 3x^2 \quad g' = e^x$

$\frac{dy}{dx} = 3x^2(e^x) + x^3(e^x)$

$= x^2 e^x (3 + x)$

b) $y = e^{x^2}$

2 layers: $e^u \quad x^2$

$\frac{dy}{dx} = [e^{x^2}][2x]$

$= 2x e^{x^2}$

c) $y = x^5 e^{x^5}$

$f = x^5 \quad g = e^{x^5}$

$f' = 5x^4 \quad g' = [e^{x^5}][5x^4]$

$\frac{dy}{dx} = 5x^4(e^{x^5}) + x^5(e^{x^5})(5x^4)$

$\frac{dy}{dx} = 5x^4 e^{x^5}(1 + x^5)$

2. Find the absolute maximum value of the function $f(x) = xe^{-x}$

I. $\frac{dy}{dx} = 0$

CN $x = 1$

$f = x \quad g = e^{-x}$

$f' = 1 \quad g' = [e^{-x}][-1]$

$f'(x) = (1)(e^{-x}) + (x)(-e^{-x})$

$f'(x) = e^{-x} - xe^{-x}$

$0 = e^{-x}(1 - x)$

$e^{-x} = 0 \qquad 1 - x = 0$

$\frac{1}{e^x} = 0 \qquad x = 1$

No Roots

II. Justify max/min

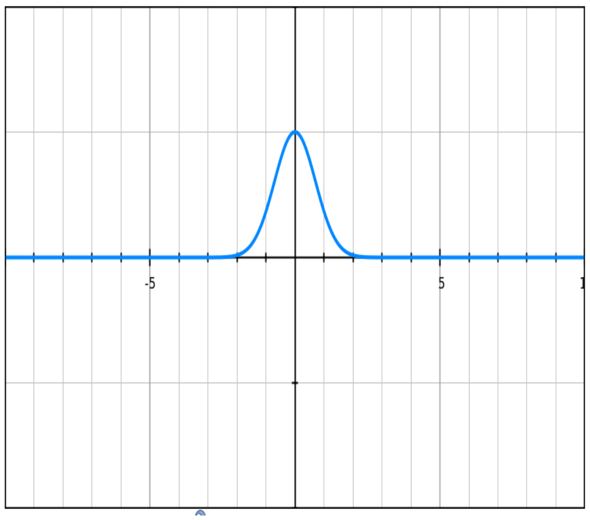
	$\frac{1}{e^x} - \frac{x}{e^x}$	$f'(x)$	$f(x)$
$(-\infty, 1)$	+	+	INC
$(1, \infty)$	−	−	DEC

MAX $x = 1$

MAX ... $f(1) = (1)(e^{-1}) = \frac{1}{e}$

3. Sketch the graph of $f(x) = e^{-x^2}$

GraphSketch.com



Homework: Page 366 # 1, 4 (a,b,d,g,h,k,l) , 5, 8, 10, 11(a,b)