

# Derivative of the Exponential Function With Base $e$

**Objectives:** Find the derivative of exponential functions.

**Warm up:** Estimations of  $e$

If you began walking at 1 km/h and then doubled your speed over a one-minute interval, you would be walking at 2 km/h. But suppose you increased your speed by 50% every half-minute. How fast would you be walking at the end of one minute?

Suppose you increased your speed by 25% every quarter-minute. What would your speed be at the end of one minute? Remember, your speed would be 1.25 times as fast every quarter-minute. Complete the chart below



Time Elapsed (s)	0	15	30	45	60
Speed (km/h)					

Generate an expression to find your speed at the end of one minute.

Suppose you increased your speed by  $\frac{1}{10}$  for every tenth of a minute. What would your speed be at the end of one minute?

Complete the table for each increase in speed for an equal portion of a minute.

Increase in speed	Speed at the end of 1 minute
$\frac{1}{10}$	
$\frac{1}{1000}$	
$\frac{1}{100000}$	
$\frac{1}{10000000}$	
$\frac{1}{1000000000}$	

# Derivatives of $y = e^x$

$e$  can be defined as:  $e = \lim_{n \rightarrow \infty} \left[ 1 + \frac{1}{n} \right]^n$

Investigate:

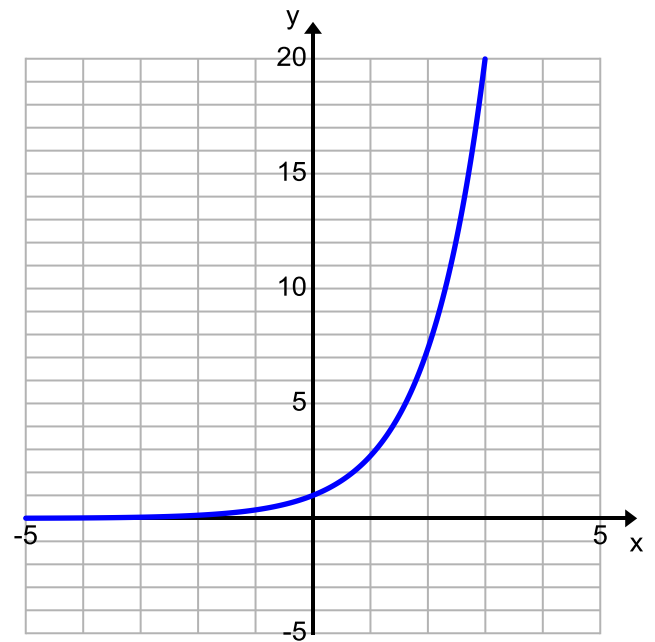
**Why is  $e$  such a special number?**

Use your calculator to sketch  $y = e^x$ .

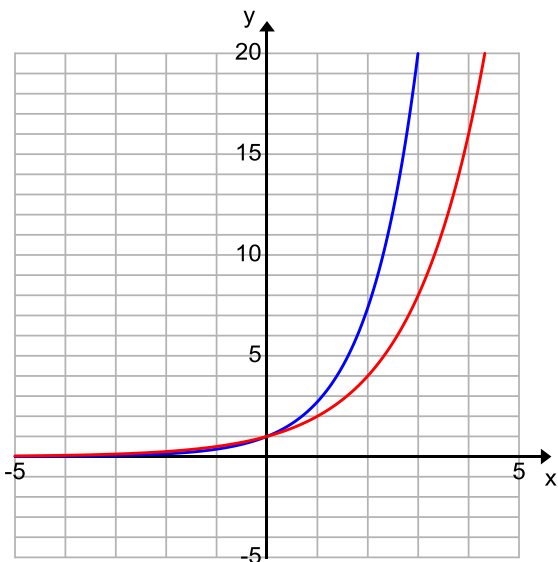
Find the values of  $e^x$  at  $x = 1, 3, 5$

Find the derivative of  $y = e^x$  at  $x = 1, 3, 5$  using your calculator.

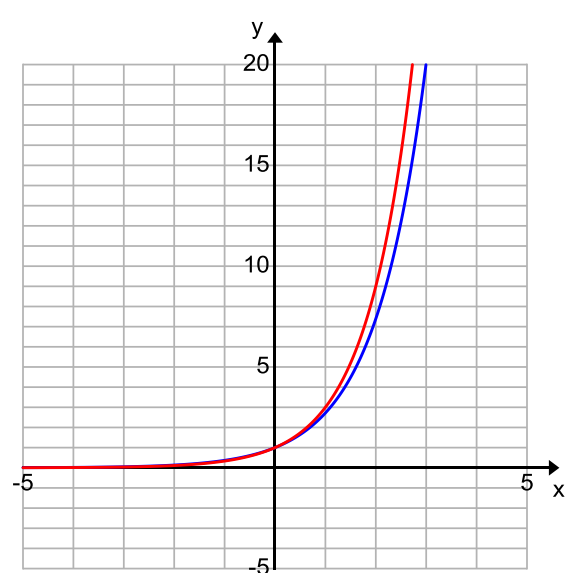
State the value of  $\frac{dy}{dx} e^x$



$y = 2^x$  and  $y = e^x$



$y = 3^x$  and  $y = e^x$



Chain Rule:  $f(x) = e^u$  then  $f'(x) = e^u \cdot \frac{du}{dx}$

1. Differentiate

a)  $y = x^3 e^x$

b)  $y = e^{x^2}$

c)  $y = x^5 e^{x^5}$

2. Find the absolute maximum value of the function  $f(x) = x e^{-x}$

3. Sketch the graph of  $f(x) = e^{-x^2}$

**Homework:** Page 366 # 1, 4 (a,b,d,g,h,k,l) , 5, 8, 10, 11(a,b)