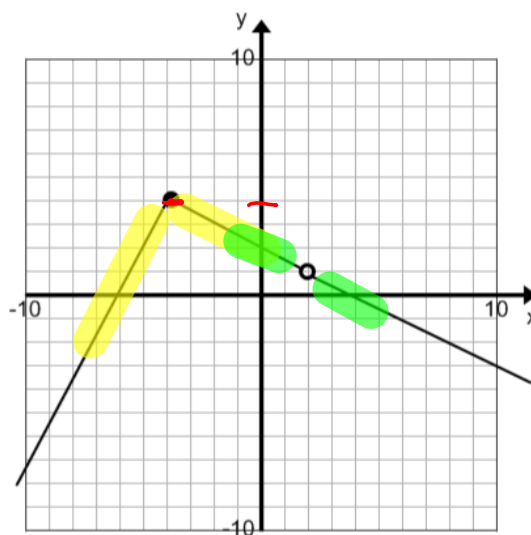


Finding the Limits of Functions – Reading Graphs

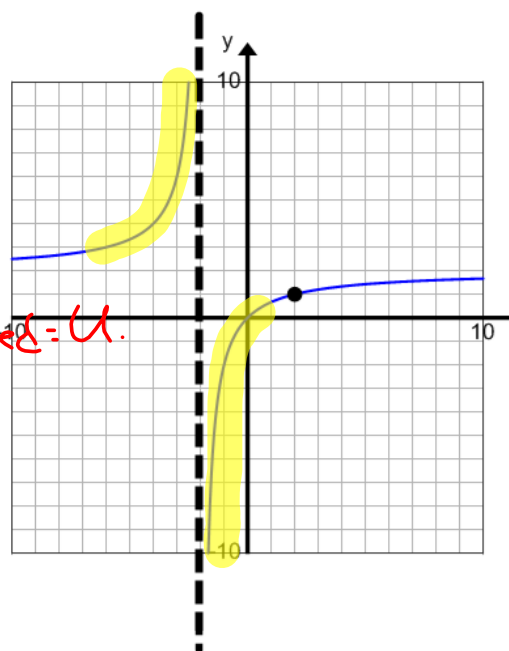
Given the function: $y = f(x)$

- Determine $\lim_{x \rightarrow -4} f(x) = 4$
- Determine $\lim_{x \rightarrow 2} f(x) = 1$



Given the function: $y = g(x)$

- Determine $\lim_{x \rightarrow -2} g(x) = \text{undefined} = U.$
- Determine $\lim_{x \rightarrow 2} g(x) = 1$



2.0 Determine Limits.2019

Finding the Limits of Functions – Algebraic Processes

Outcomes: Evaluate limit expressions in the format: $\frac{0}{0}$

1. Find $\lim_{x \rightarrow 2} \frac{4-x^2}{x-2}$

$\dots \frac{-1(x^2-4)}{(x-2)}$

Factor Theorem

$$f(x) = 4 - x^2$$

$$f(2) = 4 - (2)^2 = 0$$

$$(x-2) \text{ or } (2-x)$$

is a factor

$$= \lim_{x \rightarrow 2} \frac{\overset{(-1)}{(2-x)}(2+x)}{\cancel{(x-2)}}$$

$$= \lim_{x \rightarrow 2} -1(x+2)$$

$$= -1(2+2)$$

$$= -4$$

* keep limit until we substitute ...

2.0 Determine Limits.2019

2. Evaluate $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 3x + 2}$

$$(2)^3 - 8 = 0$$

$$\therefore (x-2)$$

Factor

$$= \lim_{x \rightarrow 2} \frac{(x-2)(x^2 + 2x + 4)}{(x-2)(x-1)}$$

$$= \lim_{x \rightarrow 2} \frac{x^2 + 2x + 4}{x-1}$$

$$= \frac{(2)^2 + 2(2) + 4}{2-1}$$

$$4+4+4$$

$$= 12$$

2.0 Determine Limits.2019

3. Find $\lim_{h \rightarrow 0} \frac{(2+h)^2 - 4}{h}$

$$(2+0)^2 - 4 = 0$$

$$(2+h)(2+h) - 4 \\ = 4 + 2h + 2h + h^2 - 4$$

$$= \lim_{h \rightarrow 0} \frac{4 + 4h + h^2 - 4}{h}$$

$$= \lim_{h \rightarrow 0} \frac{4h + h^2}{h}$$

...

$$\frac{4h}{h} + \frac{h^2}{h} \quad \text{OR} \quad \frac{h(4+h)}{h}$$

$$= \lim_{h \rightarrow 0} 4 + h$$

$$= 4$$

2.0 Determine Limits.2019

4. Find $\lim_{h \rightarrow 0} \frac{(1-h)^2 - 1}{h}$

$$= \lim_{h \rightarrow 0} \frac{1 - 2h + h^2 - 1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-2h + h^2}{h}$$

...

$$\frac{h(-2+h)}{h} \quad \text{or} \quad \frac{-2h}{h} + \frac{h^2}{h}$$

$$= \lim_{h \rightarrow 0} -2 + h$$

$$= -2 + 0$$

$$= -2$$

2.0 Determine Limits.2019

$$5. \text{ Find } \lim_{x \rightarrow 0} \frac{\sqrt{x+1}-1}{x} \left(\frac{\sqrt{x+1} + 1}{\sqrt{x+1} + 1} \right)$$

$$= \lim_{x \rightarrow 0} \frac{(x+1) - 1}{x(\sqrt{x+1} + 1)}$$

$$= \lim_{x \rightarrow 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{x+1} + 1)}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+1} + 1}$$

$$= \frac{1}{\sqrt{0+1} + 1}$$

$$= \frac{1}{2}$$

2.0 Determine Limits.2019

$$6. \text{ Find } \lim_{x \rightarrow 0} \frac{\sqrt{x+4}-2}{x} \left(\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} \right)$$

$$= \lim_{x \rightarrow 0} \frac{(x+4) - 4}{x(\sqrt{x+4}+2)}$$

$$= \lim_{x \rightarrow 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{x+4}+2)}$$

$$= \frac{1}{\sqrt{0+4}+2} \dots \frac{1}{\sqrt{4}+2} = \frac{1}{2+2}$$

$$= \frac{1}{4}$$

2.0 Determine Limits.2019

7. Find $\lim_{x \rightarrow 3} \frac{\frac{1}{x} - \frac{1}{3}}{x-3}$ $\left[\frac{3x}{3x} \right]$



Common denominator?

$$= \lim_{x \rightarrow 3} \frac{3-x}{(3x)(x-3)}$$

$$\frac{3x}{x} - \frac{3x}{3}$$

$$= \lim_{x \rightarrow 3} \frac{-1}{3x}$$

$$= -\frac{1}{3(3)}$$

$$= -\frac{1}{9}$$

2.0 Determine Limits.2019

8. Find $\lim_{x \rightarrow 0} \frac{4 - 2(x+2)}{x(x+2)}$

$$= \lim_{x \rightarrow 0} \frac{4 - 2(x+2)}{x(x+2)}$$

$$= \lim_{x \rightarrow 0} \frac{4 - 2x - 4}{x(x+2)}$$

$$= \lim_{x \rightarrow 0} \frac{-2\cancel{x}}{\cancel{x}(x+2)}$$

$$= \frac{-2}{0+2} \quad \dots \quad \frac{-2}{2}$$

$$= -1$$

2.0 Determine Limits.2019

9. Find $\lim_{x \rightarrow 4} \frac{x^{-2} - 4^{-2}}{x - 4}$

$$= \lim_{x \rightarrow 4} \frac{\frac{1}{x^2} - \frac{1}{4^2}}{x - 4} \cdot \frac{(16x^2)}{(16x^2)}$$

$$= \lim_{x \rightarrow 4} \frac{16 - x^2}{16x^2(x - 4)}$$

$$= \lim_{x \rightarrow 4} \frac{(4 - x)(4 + x)}{16x^2(x - 4)}$$

$$= \lim_{x \rightarrow 4} \frac{-1(x + 4)}{16x^2}$$

$$= \frac{-1(4 + 4)}{16(4)^2}$$

$$= -\frac{1}{32}$$

...

$$\frac{-1(\cancel{8})}{(16)(\cancel{16})} = \frac{-1}{16(2)}$$