

2.0 Find constant

Solving Differential Equations

Warm up - finding derivatives, forward thinking.

1. Find the slope function (derivative) for any point on the given curves:

a) $y = x^2 + 2x - 3$

b) $y = 3e^{2x}$

c) $y = \sin^2 x$

$$\frac{dy}{dx} = 2x + 2$$

$$y' = 3e^{2x} (2)$$

$$y' = 6e^{2x}$$

$$\frac{d}{dx}(\sin x)$$

$$\frac{dy}{dx} = 2 \sin x (\cos x)$$

$$\frac{dy}{dx} = \sin(2x)$$

2. Find the slope of the above curve at the point where

a) $x = 3$

b) $x = 3$

c) $x = \frac{\pi}{4}$

$$f'(3) = 2(3) + 2$$

$$f'(3) = 6e^6$$

$$f'(x) = \sin(2 \cdot \frac{\pi}{4})$$

$$m = 8$$

$$P(3, 12)$$

$$= \sin \frac{\pi}{2}$$

$$= 1$$

Outcome: Find the equation for the antiderivative given initial conditions.

SKILL: Work backwards. Solve for C given a point on the function.

Examples:

1. A curve has a general slope described by $2x - 5$. If the original curve passes through the point $(2, 17)$, then what is the equation of the original curve?

$$f(x) = 2x - 5$$

$$F(x) = \frac{2x^2}{2} - \frac{5x}{1} + c$$

$$F(x) = x^2 - 5x + c$$

Point $(2, 17)$ $x = 2$ $y = 17$

$$17 = (2)^2 - 5(2) + c$$

$$c = 23$$

$$F(x) = x^2 - 5x + 23$$

2. Find the equation of each curve:

a) $f(x) = \frac{6}{x^2}$ and passing through $(-1, 6)$.

$$f(x) = 6x^{-2}$$

$$F(x) = -\frac{6}{x} + c$$

$$F(x) = \frac{6x^{-1}}{-1} + c$$

$$P(-1, 6)$$

$$6 = -\frac{6}{-1} + c$$

$$6 = 6 + c$$

$$c = 0$$

$$F(x) = -\frac{6}{x}$$

b) $f(x) = 3\sqrt{x}$ and passing through $(4, 5)$

$$f(x) = 3x^{1/2}$$

$$F(x) = \frac{3x^{3/2}}{3/2} + c$$

$$F(x) = \frac{2}{3}(3x^{3/2}) + c$$

$$F(x) = 2x^{3/2} + c \quad P(4, 5)$$

$$5 = 2(\sqrt{4})^3 + c$$

$$5 = 2(2)^3 + c$$

$$5 = 16 + c$$

$$c = -11$$

2.0 Find constant

$$y = 4 \sin x$$

c) $f(x) = 4 \sin$ and passing through $(\frac{\pi}{2}, 6)$

$$F(x) = -4 \cos x + C$$

$$6 = -4 \left[\cos \frac{\pi}{2} \right] + C$$

$$6 = -4(0) + C$$

$$C = 6$$

$$F(x) = -4 \cos x + 6$$

2. Find the displacement function for an object moving on a horizontal line given the

velocity function: $\frac{ds}{dt} = 2t$, with the initial condition: $s = 3$ when $t = 0$.

$$s'(t) = 2t$$

$$s(t) = \frac{2t^2}{2} + C$$

$$s(t) = t^2 + C$$

$$3 = (0)^2 + C$$

$$C = 3$$

$$s(t) = t^2 + 3$$

3. Find the curve $y = F(x)$ that passes through $(-1, 0)$ and satisfies $\frac{dy}{dx} = 6x^2 + 6x$

$$F(x) = \frac{6x^3}{3} + \frac{6x^2}{2} + C$$

$$F(x) = 2x^3 + 3x^2 + C$$

$$P(-1, 0)$$

$$0 = 2(-1)^3 + 3(-1)^2 + C$$

$$0 = -2 + 3 + C$$

$$C = -1$$

$$F(x) = 2x^3 + 3x^2 - 1$$

4. For the graph G at every point $\frac{dy}{dx} = e^{-x}$ Find the equation of a graph parallel to G that passes through the origin.

$$F(x) = \frac{e^{-x}}{-1}$$

$$y = -e^{-x} + C$$

$$P(0, 0)$$

$$0 = -e^{-(0)} + C$$

$$0 = -(e^0) + C$$

$$0 = -1 + C$$

$$C = 1$$

$$y = -\frac{1}{e^x} + 1$$

Homework: Page 411 #1, 2, 3, 4abc, 5, 6