6. First Test.

The First Derivative Test

The point where a function changes from:

Increasing to Decreasing is a local and/or absolute maximum.

Decreasing to Increasing is a local and/or absolute minimum.

Outcome: Identify the Maximum and Minimum of functions using the first derivative test.

1. a) State the interval in which the following function is increasing and decreasing.

   $y = x^3 + 6x^2 + 9x + 2$

   b) State the value where the first derivative is equal to zero.

   c) How can you use intervals of increase and decrease to determine if you have a max or min?

Critical Numbers $f'(x) = 0$ $f'(x) = U.$

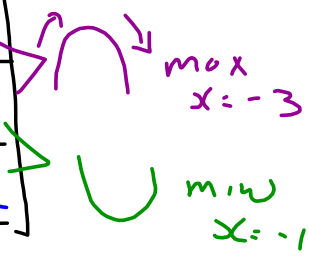
$\frac{dy}{dx} = 3x^2 + 12x + 9$

$0 = 3(x^2 + 4x + 3)$

$0 = (x+3)(x+1)$

$x = -3$ $x = -1$

	$x+3$	$x+1$	slope $f'(x)$	$f(x)$
$(-\infty, -3)$	neg	neg	pos	INC
$(-3, -1)$	pos	neg	NEG	DEC
$(-1, \infty)$	pos	pos	Pos	INC



6. First Test.

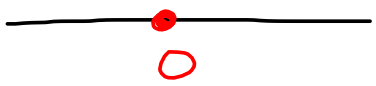
2. a) State the interval in which the following function is increasing and decreasing. $y = x^3$
   b) State the value where the first derivative is equal to zero in the function $y = x^3$.
   c) Is this value a maximum or minimum?

$y = x^3$

$\frac{dy}{dx} = 3x^2$

$0 = 3x^2$

$x = 0$



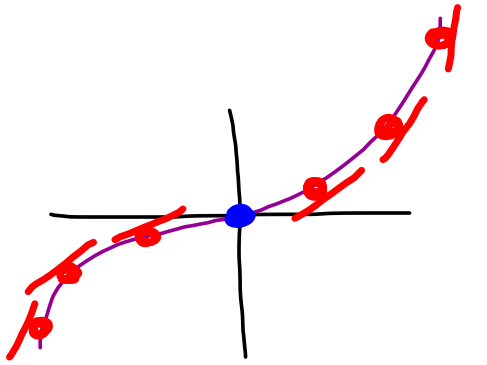
	x^2	$f'(x)$	$f(x)$
$(-\infty, 0)$	pos	pos	INC
$(0, \infty)$	pos	pos	INC

OR

	x	x	$f'(x)$	$f(x)$
$(-\infty, 0)$	−	−	+	INC
$(0, \infty)$	+	+	+	INC

MAX... inc to dec

Min... dec to inc



$x = 0$

$y = x^3$

$y = 0$

Point $(0, 0)$ is a point of inflection.

6. First Test.

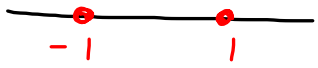
3. Find the maximums and minimums for the following functions. Are they local or absolute max/min values? Sketch the graphs.

a) $f(x) = x^3 - 3x + 1$

$f'(x) = 3x^2 - 3$

$0 = 3(x^2 - 1)$

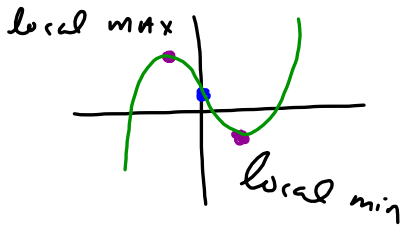
$0 = (x+1)(x-1)$



	$x+1$	$x-1$	slopes $f'(x)$	$f(x)$
$(-\infty,-1)$	neg	neg	pos	INC
$(-1,1)$	pos	neg	neg	DEC
$(1,\infty)$	pos	pos	pos	INC

max $f(-1) = (-1)^3 - 3(-1) + 1 = 3$

min $f(1) = (1)^3 - 3(1) + 1 = -1$

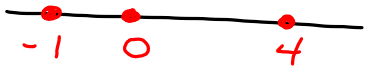


b) $y = x^4 - 4x^3 - 8x^2 - 1$

$\frac{dy}{dx} = 4x^3 - 12x^2 - 16x$

$0 = 4x(x^2 - 3x - 4)$

$0 = 4x(x-4)(x+1)$



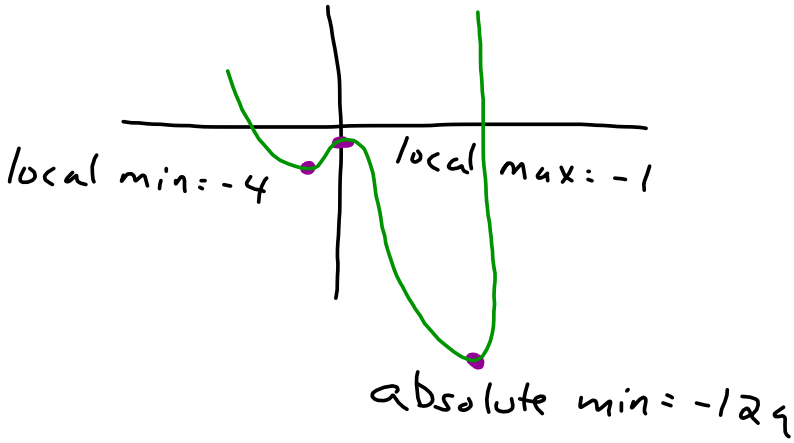
$f(x) = x^4 - 4x^3 - 8x^2 - 1$

	$x+1$	x	$x-4$	slope $f'(x)$	$f(x)$
$(-\infty,-1)$	−	−	−	−	DEC
$(-1,0)$	+	−	−	+	INC
$(0,4)$	+	+	−	−	DEC
$(4,\infty)$	+	+	+	+	INC

min $f(-1) = -4$

max $f(0) = -1$

min $f(4) = -129$



6. First Test.

c) $f(x) = 2 - 2x^{-\frac{1}{3}}$

$f(x) = 2 - \frac{2}{\sqrt[3]{x}}$

$x \neq 0$

$f'(x) = \frac{2}{3} x^{-4/3}$

$0 = \frac{2}{3\sqrt[3]{x^4}}$

no point where $f'(x) = 0$

but $f'(x) = U$ where $x = 0$

$\frac{2}{3\sqrt[3]{x^4}}$

	$\sqrt[3]{x^4}$	slope $f'(x)$	$f(x)$
$(-\infty, 0)$	+	pos	inc
$(0, \infty)$	+	pos	inc

no max/min ...

6. First Test.

4. Find the absolute maximum and minimum values of the function.

$f(x) = x^3 + 6x^2 + 9x + 2$, in $\quad -3.5 \le x \le 1$

$x = -3.5$
$x = 1$] critical numbers

$f'(-3.5) = U$

$f'(1) = U$

$\frac{dy}{dx} = 3x^2 + 12x + 9$

$0 = 3(x^2 + 4x + 3)$

$0 = (x+3)(x+1)$

$x = -3 \quad x = -1$

	$x+3$	$x+1$	slope $f'(x)$	$f(x)$
$(-\infty,-3)$	neg	neg	pos	INC
$(-3,-1)$	pos	neg	NEG	DEC
$(-1,\infty)$	pos	pos	Pos	INC

max $f(-3) =$

min $f(-1) =$

4 critical numbers ... $f(x) = x^3 + 6x^2 + 9x + 2$

$f(-3.5) =$ $\qquad$ $f(-3) =$

$f(1) =$ $\qquad$ $f(-1) =$