2.0 Limits for slopes

touch at one pt   cross at 2 pts

Objective:

Find the tangent slope to a curve using secants.
Find the tangent slope to a curve using limits.
Evaluate limits.

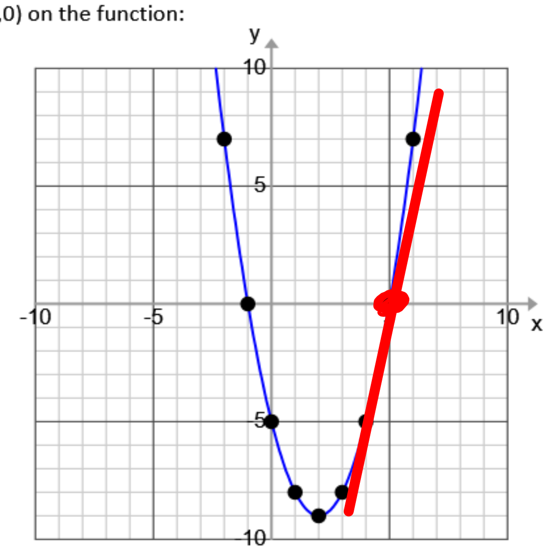
SKILLS Review: Define slope using words, symbols and formulas.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example: Find the slope at P(5,0) on the function:

$y = x^2 - 4x - 5$

$y = 4.9^2 - 4(4.9) - 5$



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Plan A:

Find the slopes of secant lines by using points B(x,y) from both the left and right side of point A(5,0).
Predict the tangent slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

		secant slope - cross curve at two points'
Point 1	**Point 2**	**slope of AB**
A (5,0)	B (3,-8)	m= 4
A (5,0)	B (4,-5)	m= 5
A (5,0)	B(4.5,-2.75)	m= 5.5
A (5,0)	B (4.9, y)	m= 5.9
A (5,0)	B (4.99, y)	m= 5.99
(5,0) (5,0) Predict tangent slope at A, as you approach from left side of A.		m= 6

$m = \frac{0}{0}$

		secant slope - cross curve at two points'
Point 1	**Point 2**	**slope of AB**
A (5,0)	B (6,7)	m= 7
A (5,0)	B (5.5,3.25)	m= 6.5
A (5,0)	B (5.1, y)	m= 6.1
A (5,0)	B(5.01, y)	m= 6.01
(5,0) (5,0) Predict tangent slope at A, as you approach from right side of A.		m= 6

$\frac{0}{0}$

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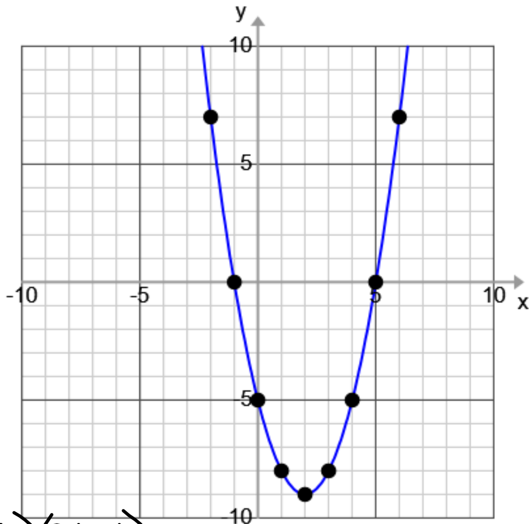
Plan B: Use algebraic limits to find the tangent slope.



secant $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
2 pts

tangent $m = \dfrac{y - y_1}{x - x_1} = \dfrac{0}{0}$
1 pt

$m = \lim_{x \to 5} \dfrac{y - y_1}{x - x_1}$

$m = \lim_{x \to 5} \dfrac{(x^2 - 4x - 5) - 0}{(x) - 5}$

$m = \lim_{x \to 5} \dfrac{x^2 - 4x - 5}{x - 5} = \lim_{x \to 5} \dfrac{(x-5)(x+1)}{(x-5)}$

$= \lim_{x \to 5} (x+1)$ "evaluate"

$m = 5 + 1$
$m = 6$

Application/Skills:

- Change a slope equation to a limit statement, and evaluate the limit to find tangent slope:
  Continuing with the quadratic function, $y = x^2 - 4x - 5$ we can find the slope to the parabola at any point by writing limit statements and evaluating:
  a) P(-2,7)
  b) P(4,-5)

- Evaluate Limits:
  $\lim_{x \to 3} \frac{x^2 + 2x - 15}{x - 3}$

2.0 Limits for slopes

Continuing with the quadratic function, $y = x^2 - 4x - 5$ we can find the slope to the parabola at any point by writing limit statements and evaluating:

a) P(-2,7)
b) P(4,-5)

$y = x^2 - 4x - 5$

Evaluate Limits:

a) $m = \frac{y_2 - y_1}{x_2 - x_1}$ secant

$m = \lim_{x \to a} \frac{y - y_1}{x - x_1}$ tangent

$= \lim_{x \to -2} \frac{(x^2 - 4x - 5) - (7)}{x + 2}$

$= \lim_{x \to -2} \frac{x^2 - 4x - 12}{x + 2}$

$= \lim_{x \to -2} \frac{(x+2)(x-6)}{(x+2)}$

$m = \lim_{x \to -2} (x - 6)$

$m = -2 - 6$

$m = -8$

b) $P(4,-5)$ $\quad y = x^2 - 4x - 5$

tangent $m = \lim_{x \to 4} \frac{y - y_1}{x - x_1}$

$m = \lim_{x \to 4} \frac{(x^2 - 4x - 5) + 5}{x - 4}$

$= \lim_{x \to 4} \frac{x^2 - 4x}{x - 4}$

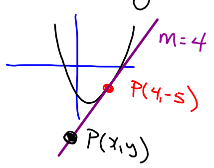
$= \lim_{x \to 4} \frac{x(x-4)}{(x-4)}$

$m = \lim_{x \to 4} x$

$m = 4$

EXTRA

Find the equation of the tangent line on $y = x^2 - 4x - 5$ at $(4,-5)$.



$m = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{4}{1} = \frac{y + 5}{x - 4}$

$4x - 16 = y + 5$

$4x - y = 21$

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- Evaluate Limits:

$$\lim_{x\to 3}\frac{x^2+2x-15}{x-3} = \lim_{x\to 3}\frac{(x-3)(x+5)}{(x-3)}$$

$$= \lim_{x\to 3}(x+5)$$

$$= 3+5$$

$$= 8$$