2.2 Product Rule

The Product Rule

Given: $y = (2x+1)(3x+2)$ $= 6x^2+4x+3x+2$

- **Expand to find the derivative.**

$y = 6x^2+7x+2$

$\frac{dy}{dx} = 12x+7$

- **Find the derivative of** $f = 2x+1$
- **Find the derivative of** $g = 3x+2$

$f(x) = 2x+1$ $\quad g(x) = 3x+2$

$f' = 2$ $\quad g' = 3$

Given: $y = (5x-1)(4x^2+3)$ $= 20x^3+15x-4x^2-3$

- **Expand to find the derivative.**

$y = 20x^3-4x^2+15x-3$

$\frac{dy}{dx} = 60x^2-8x+15$

- **Find the derivative of** $f = 5x-1$
- **Find the derivative of** $g = 4x^2+3$

$f = 5x-1$ $\quad g = 4x^2+3$

$f' = 5$ $\quad g' = 8x$

Given: $y = (x^8)(x^3)$

- **Expand to find the derivative.**

$y = x^{11}$ $\quad \frac{dy}{dx} = 11x^{10}$

- **Find the derivative of** $f = x^8$
- **Find the derivative of** $g = x^3$

$f = x^8$ $\quad g = x^3$

$f' = 8x^7$ $\quad g' = 3x^2$

Pattern ?

2.2 Product Rule

Rule:

$y = [f][g]$

$$\frac{dy}{dx} = f'g + fg'$$

Examples:
Page 92 – 1ac, 2bc,3e

Practice:
Page 92 – 2adg, 3acf

EXERCISE 2.4

1. Use the Product Rule to find the derivative. Do not simplify your answer.
   (a) $f(x) = (2x - 1)(x^2 + 1)$ (b) $f(x) = x(3x - 8)$
   (c) $y = x^2(1 + x - 3x^2)$ (d) $y = (x^3 + x^2 + 1)(x^2 + 2)$
   (e) $f(t) = (t^4 + t^2 - 1)(t^2 - 2)$ (f) $f(t) = \sqrt[3]{t}(1 - t)$
   (g) $F(y) = \sqrt{y}(y - 2\sqrt{y} + 2)$ (h) $G(y) = (y - y^2)(2y - y^{\frac{4}{3}})$
2. Use the Product Rule to differentiate each function. Simplify your answer.
   (a) $y = x^3(x^2 + 2x + 3)$ (b) $y = x^{-2}(x^3 - 3x^2 + 6)$
   (c) $f(x) = (1 - x^2)(2 - x^3)$ (d) $f(x) = (3x^3 + 4)(1 - 2x^3)$
   (e) $f(t) = (6 + t^{-2})(8t^{10} - 5t^3)$ (f) $f(t) = (at + b)(ct^2 - d)$
   (g) $g(u) = \sqrt{u}(2 - u^2 + 5u^4)$ (h) $g(v) = (v - \sqrt{v})(v^2 + \sqrt{v})$
3. Find the slope of the tangent to the given curve at the point whose x-coordinate is given.
   (a) $y = (1 - 2x)(3x - 4), x = 2$
   (b) $y = (1 - x + x^2)(x - 2), x = 1$
   (c) $y = x^4(4x^3 + 2), x = -1$
   (d) $y = (1 + x - 2x^2)(3x^3 + x - 1), x = 1$
   (e) $y = x^{-5}(1 + x^{-1}), x = 1$
   (f) $y = (2 - 3\sqrt{x})(4 - \sqrt{x}), x = 4$

1a) $f(x) = (2x-1)(x^2+1)$

$f = 2x-1 \qquad g = x^2+1$

$f' = 2 \qquad g' = 2x$

$f'(x) = f'g + fg'$

$f'(x) = 2(x^2+1) + 2x(2x-1)$

$= 2x^2 + 2 + 4x^2 - 2x$

$= 6x^2 - 2x + 2$

3e) $y = x^{-5}(1+x^{-1})$, $x=1$

$f = x^{-5}$ $\quad g = 1+x^{-1}$

$f' = -5x^{-6}$ $\quad g' = -1x^{-2}$

$$\frac{dy}{dx} = f'g + fg'$$

$$\frac{dy}{dx} = (-5x^{-6})(1+x^{-1}) + (x^{-5})(-1x^{-2})$$

$$= -5x^{-6} - 5x^{-7} - x^{-7}$$

$$= \frac{-5x^{-6} - 6x^{-7}}{1}\cdot\frac{x^7}{x^7} \text{ or } -\frac{5}{x^6} - \frac{6}{x^7}$$

$$= \frac{-5x-6}{x^7}$$

$$f'(1) = \frac{-5(1)-6}{(1)^7} = -11$$