2.4 Chain Advanced

Differentiation Using a Combination of Rules

'Simpler Rule' Differentiation.

Find derivatives for each of the following:

a) $y = 251x^{100}$

b) $y = 3x^2 + 2x$

a) $\frac{dy}{dx} = 25100x^{99}$

b) $\frac{dy}{dx} = 6x + 2$

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c) $y = (5x^2 - 3)(2x^3 + 1)$ practice your product rule with this one.

$f = 5x^2 - 3$

$f' = 10x$

$g = 2x^3 + 1$

$g' = 6x^2$

$$\frac{dy}{dx} = f'g + fg'$$

$$\frac{dy}{dx} = 10x(2x^3+1) + 6x^2(5x^2-3)$$

$$= 20x^4 + 10x + 30x^4 - 18x^2$$

$$= 50x^4 - 18x^2 + 10x$$

d) $y=\dfrac{4x^3+1}{x^2-2}$

$f=4x^3+1$ $\qquad g=x^2-2$

$f'=12x^2$ $\qquad g'=2x$

$$\frac{dy}{dx}=\frac{f'g-fg'}{g^2}$$

$$\frac{dy}{dx}=\frac{12x^2(x^2-2)-2x(4x^3+1)}{(x^2-2)^2}$$

$$=\frac{2x\left[6x(x^2-2)-1(4x^3+1)\right]}{(x^2-2)^2}$$

$$=\frac{2x\left[6x^3-12x-4x^3-1\right]}{(x^2-2)^2}$$

$$=\frac{2x(2x^3-12x-1)}{(x^2-2)^2}$$

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e) $y = (2x^5 - 5)^3$

$$\frac{dy}{dx} = 3(2x^5 - 5)^2 \frac{d}{dx}(2x^5 - 5)$$

$$\frac{dy}{dx} = 3(2x^5 - 5)^2 (10x^4)$$

$$= 30x^4(2x^5 - 5)^2$$

"Multiple Skilled" Differentiation – Chain Rule Advanced.

Differentiate the following:

a) $y=\left(\frac{2x-1}{x+2}\right)^6$ ∘∘∘ $y=[f(x)]^6$

$$\frac{dy}{dx}=6\left[\frac{2x-1}{x+2}\right]^5 \cdot \frac{d}{dx}\left(\frac{2x-1}{x+2}\right)$$

$f=2x-1$ $\quad$ $g=x+2$

$f'=2$ $\quad$ $g'=1$

$$\frac{f'g-fg'}{g^2}$$

$$=\frac{2(x+2)-1(2x-1)}{(x+2)^2}$$

$$=\frac{2x+4-2x+1}{(x+2)^2}$$

$$\frac{dy}{dx}=6\left[\frac{2x-1}{x+2}\right]^5\left[\frac{5}{(x+2)^2}\right]$$

$$=\frac{30(2x-1)^5}{(x+2)^7}$$

b) $y=(x^2+1)^3(2-3x)^4$

$f=(x^2+1)^3 \qquad g=(2-3x)^4$

$\frac{dy}{dx}=f'g+fg'$

$f'=3(x^2+1)^2(2x) \qquad g'=4(2-3x)^3(-3)$

$f'=6x(x^2+1)^2 \qquad g'=-12(2-3x)^3$

$\frac{dy}{dx}=6x(x^2+1)^2(2-3x)^4-12(2-3x)^3(x^2+1)^3$

$=6(x^2+1)^2(2-3x)^3\left[x(2-3x)-2(x^2+1)\right]$

$\frac{dy}{dx}=6(x^2+1)^2(2-3x)^3(-5x^2+2x-2)$

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c) $y=\sqrt{x+\sqrt{x^2+1}}$

$$y = \left[x + (x^2+1)^{1/2}\right]^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[x + (x^2+1)^{1/2}\right]^{-1/2} \cdot \frac{d}{dx}\left[x + (x^2+1)^{1/2}\right]$$

$$\frac{d}{dx}[x] + \frac{d}{dx}\left[x^2+1\right]^{\boxed{1/2}}$$

$$1 + \frac{1}{2}\left[x^2+1\right]^{-1/2} \cdot \frac{d}{dx}(x^2+1)$$

$$1 + \frac{1}{2}(x^2+1)^{-1/2}(2x)$$

$$1 + x(x^2+1)^{-1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[x + (x^2+1)^{1/2}\right]^{-1/2}\left[1 + x(x^2+1)^{-1/2}\right]$$

$$\frac{dy}{dx} = \frac{1}{2\left[x + (x^2+1)^{1/2}\right]^{1/2}} \cdot \left[1 + \frac{x}{(x^2+1)^{1/2}}\right]$$

$$\left[\frac{(x^2+1)^{1/2} + x}{(x^2+1)^{1/2}}\right]$$

$$= \frac{\sqrt{x^2+1} + x}{2\sqrt{x^2+1}\sqrt{x+\sqrt{x^2+1}}}$$

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Practice

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… and more if you have time.

6. Differentiate:

(a) $F(x) = x\sqrt{x^2 + 1}$ (b) $F(x) = (2x + 1)(4x - 1)^5$

(c) $G(x) = (x^2 - 1)^4(2 - 3x)$

(d) $G(x) = (x^4 - x + 1)^2(x^2 - 2)^3$

(e) $F(x) = \dfrac{x}{\sqrt{2x + 3}}$ (f) $f(t) = \dfrac{(1 + 2t)^5}{(3t^2 - 5)^2}$

(g) $g(x) = \left(\dfrac{x + 2}{x - 2}\right)^3$ (h) $h(t) = \left(\dfrac{t^2 + 1}{t + 1}\right)^{10}$

(i) $y = \sqrt{\dfrac{x^2 - 1}{x^2 + 1}}$ (j) $y = \dfrac{(2x + 3)^3}{\sqrt{4x - 7}}$

(k) $y = 3\sqrt{x}(2x + 1)^5 + \sqrt{4x - 3}$

(l) $y = \sqrt{1 + \sqrt[3]{x}}$

(m) $y = (t + \sqrt[3]{t + t^2})^{20}$ (n) $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

Stewart J. (1989) *Calculus: a first course*. McGraw-Hill Ryerson Limited.