Implicit Differentiation

How do you find the derivative when ‘y’ cannot be isolated?

$$P(x) = fg \text{ then } P'(x) = f'g + fg'$$

$$Q(x) = \frac{f}{g} \text{ then } Q'(x) = \frac{f'g - fg'}{g^2}$$

$$F(x) = \left[f(x) \right]^n, \text{ then } F'(x) = n\left[f(x) \right]^{n-1} f'(x)$$

Find the following derivatives:

$y = \left[f(x) \right]^3$

$y = x^3$

$16 = x^2 + \left[f(x) \right]^2$

$16 = x^2 + y^2$

$4x^2 + 9\left[f(x) \right]^2 = 36 \quad or \quad 4x^2 + 9y^2 = 36$

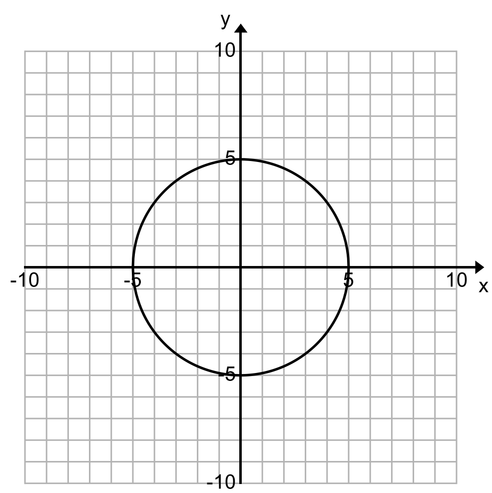
Pattern: Every time we have *'y'* in an equation, we need to write its derivative the same way we would write the derivative if we were using function notation. This pattern of differentiation is called 'Implicit Differentiation'. In all of our basic functions, we did this as part of the prescribed notation:

$$\frac{d}{dx}[y = x^2 + x - 6] \rightarrow \frac{d}{dx}[y] = \frac{d}{dx}[x^2] + \frac{d}{dx}[x] - \frac{d}{dx}6$$

$$\boxed{\frac{d}{dx}[y] = \frac{dy}{dx}} \qquad \frac{dy}{dx} = 2x + 1$$

Apply the rule to find tangent slopes, to differentiate:

1. The relation $x^2 + y^2 = 25$ is a circle centered at the origin with a radius of 5.



Find slope for each line tangent at $x = 3$

Draw tangent lines (two of them) to the circle at $x = 3$.

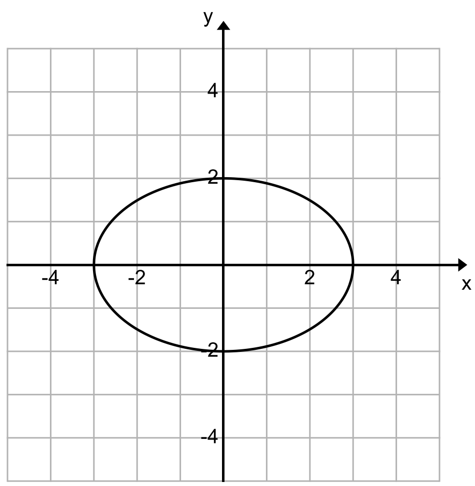
2. Differentiate $y^2 = x$

3. Determine $\frac{dy}{dx}$ for $2x^3 + y^2 - 2x^2 = 0$

4. Determine $\frac{dy}{dx}$ for $y^4 + y^3 = x^5 + x$

5. Determine y' for $2y^3 + y^2 - 2x^2 = 0$

6. Find the slope of the tangent lines to the graph $\frac{x^2}{9} + \frac{y^2}{4} = 1$, when $x = 1$



7. Determine y' for $2x^3 + x^2y^2 - 2y^2 = 0$

Homework: Page 107 #1 – 3 (every second letter) 5, 7

Stewart J. (1989) *Calculus: a first course*. McGraw-Hill Ryerson Limited.