Higher Derivatives

Outcome: Find 2nd and 3rd degree derivatives

Warm up: Find the following derivatives:

$y = \left[\text{a) } 3x^5\right]\frac{d}{dx}$ b) $\left[15x^4\right]\frac{d}{dx}$ c) $\left[60x^3\right]\frac{d}{dx}$ d) $\left[180x^2\right]\frac{d}{dx}$

$\frac{dy}{dx} = y' = 15x^4$ $y'' = 60x^3$ $180x2$ $= 360x$

Notation: The following are common ways to write higher order derivatives

1st	$f'(x)$	$\frac{d}{dx}(f(x))$	$\frac{dy}{dx}$	y'
2nd	$f''(x)$	$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$		y''
3rd	$f'''(x)$	$\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d^3y}{dx^3}$	y'''	
4th	$f^{(4)}(x)$	$\frac{d}{dx}\left(\frac{d^3y}{dx^3}\right) = \frac{d^4y}{dx^4}$		

Examples:

1. Find $\frac{d^2y}{dx^2}$ if $y = x^6$

$y' = 6x^5$

$y'' = 30x^4$

2. If $y = x^3 - 2x^2$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

$$\frac{dy}{dx} = 3x^2 - 4x$$

$$y'' = 6x - 4$$

3. Find the second derivative of $f(x) = 5x^2 + \sqrt{x}$

$$f(x) = 5x^2 + x^{1/2}$$

$$f'(x) = 10x + \frac{1}{2}x^{-1/2}$$

$$f''(x) = 10 - \frac{1}{4}x^{-3/2}$$

$$= \frac{10}{1} - \frac{1}{4x^{3/2}}$$

$$= \frac{40x^{3/2} - 1}{4x^{3/2}}$$

$$= \frac{40\sqrt{x^3} - 1}{4\sqrt{x^3}}$$

4. Find $f''(1)$ if $f(x)=\left(2-x^2\right)^{10}$

$f'(x)=10(2-x^2)^9(-2x)$

$f'(x)=-20x(2-x^2)^9$

$f=-20x$

$f'=-20$

$g=(2-x^2)^9$

$g'=9(2-x^2)^8(-2x)$

$g'=-18x(2-x^2)^8$

$f''(x)=-20(2-x^2)^9+(-20x)(-18x)(2-x^2)^8$

$f''(x)=-20(2-x^2)^8\left[(2-x^2)-18x^2\right]$

$f''(x)=-20(2-x^2)^8(2-19x^2)$

$f''(x)=20(2-x^2)^8(19x^2-2)$

$f''(1)=20(1)^8(17)$

$f''(1)=340$

5. If $x^3 + y^3 = 5$ use implicit differentiation to find the second order derivative of y.

$$\frac{d}{dx}\left[x^3 + y^3 = 5\right]$$

$$\frac{d}{dx}\left[x^3\right] + \frac{d}{dx}\left[y^3\right] = \frac{d}{dx}\left[5\right]$$

$$3x^2 + 3y^2 \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-3x^2}{3y^2}$$

$$\frac{d}{dx}\left[\frac{dy}{dx} = -\frac{x^2}{y^2}\right]$$

$$\frac{d^2y}{dx^2} = y'' =$$

$f = -x^2$ $\quad g = y^2$

$f' = -2x$ $\quad g' = 2y\frac{dy}{dx}$

$$g' = 2y\left[-\frac{x^2}{y^2}\right]$$

$$g' = -\frac{2x^2}{y}$$

$$y'' = \frac{(-2x)(y^2) + (x^2)\left(-\frac{2x^2}{y}\right)}{\left[y^2\right]^2}\left[\frac{y}{y}\right]$$

$$y'' = \frac{-2xy^3 - 2x^4}{y^5}$$

$$y'' = -\frac{2x(y^3 + x^3)}{y^5}$$

... $x^3 + y^3 = 5$

$$y'' = -\frac{2x(5)}{y^5}$$

$$y'' = \frac{-10x}{y^5}$$

6. Find y'' if $y^2 - xy = 3$

$$\left[y^2 - xy = 3\right]\frac{d}{dx}$$

$$\frac{d}{dx}[y^2] - \frac{d}{dx}[xy] = \frac{d}{dx}[3]$$

$$2y\cdot\frac{dy}{dx} - \left[\begin{array}{ll} f=x & g=y \\ f'=1 & g'=1\cdot\frac{dy}{dx} \\ (1)(y)+(x)\left(\frac{dy}{dx}\right) & \end{array}\right] = 0$$

$$2y\frac{dy}{dx} - y - x\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left[2y-x\right] = y$$

$$\left[\frac{dy}{dx} = \frac{y}{2y-x}\right]\frac{d}{dx} \quad \ldots \frac{d^2y}{dx^2}$$

$f = y$ $\qquad$ $g = 2y - x$

$f' = \frac{dy}{dx}$ $\qquad$ $g' = 2\frac{dy}{dx} - 1$

$f' = \frac{y}{2y-x}$ $\qquad$ $g' = 2\left[\frac{y}{2y-x}\right] - 1$

$$= \frac{2y}{2y-x} - \left[\frac{2y-x}{2y-x}\right]$$

$$g' = \frac{x}{2y-x}$$

$$\frac{f'g - fg'}{g^2}$$

$$f''(x) = \frac{\left(\frac{y}{2y-x}\right)\left(\frac{2y-x}{1}\right) - \left(\frac{y}{1}\right)\left(\frac{x}{2y-x}\right)}{[2y-x]^2}\left[\frac{2y-x}{2y-x}\right]$$

$$f'' = \frac{2y^2 - xy - xy}{(2y-x)^3}$$

$$f'' = \frac{2y^2 - 2xy}{(2y-x)^3} = \frac{2(y^2-xy)}{(2y-x)^3}$$

$$f''(x) = \frac{2(3)}{(2y-x)^3} = \frac{6}{(2y-x)^3}$$

Homework: Page 111 # 1, 2, 4, 5
Stewart J. (1989) *Calculus: a first course*. McGraw-Hill Ryerson Limited.

EXERCISE 2.8

B **1.** Find the first and second derivatives of the given functions.

(a) $f(x) = x^5 - 4x^2 + 1$

(b) $g(x) = 7x^4 + 12x^3 - 4x + 8$

(c) $f(t) = 2t - \dfrac{1}{t+1}$ (d) $g(t) = \dfrac{4}{\sqrt{t}}$

(e) $y = (2x + 1)^8$ (f) $y = t^3 + \dfrac{1}{t^3}$

(g) $y = \sqrt{x^2 + 1}$ (h) $y = \dfrac{t}{t - 1}$

2. Find the third derivative.

(a) $f(x) = 1 - 12x + 4x^2 - x^3$ (b) $f(x) = \dfrac{1}{x^5}$

(c) $y = \dfrac{3}{(4 - x)^2}$ (d) $y = \sqrt{1 + 2x}$

3. Find the first six derivatives of the function $y = x^5 + x^4 + x^3 + x^2 + x + 1$.

4. If $f(x) = \sqrt{1 + x^3}$, find $f''(2)$.

5. If $g(x) = \dfrac{1}{\sqrt{3x + 4}}$, find $g'''(4)$.