Basic Integration

Basic Rules (we us intuitively):

1. One basic integration formula states

   $$\int af(x) = a\int f(x) \xrightarrow{\text{Further more}} a\int f(x) = a[F(x)+c]$$

   Use the above formula to help integrate $\int 13x^{45}dx$

   $= 13\int x^{45}\,dx$ we took $\frac{d}{dx}$ to find $f'(x)$

   $= 13\left[\frac{x^{46}}{46}\right] + c$

2. A second integration formula states

   $$\int x^n dx = \frac{x^{n+1}}{n+1}$$

   Use the above formula to help integrate $\int x^4 dx = \frac{x^5}{5} + c$

3. A third integration formula states:

   $$\int f(x) \pm g(x) = \int f(x) \pm \int g(x)$$

   Use the above formulas to help integrate $\int (2x^2 - 5x - 2x^{-3})dx$

   $= \int 2x^2 - \int 5x - \int 2x^{-3}$

   $= \frac{2}{3}x^3 - \frac{5}{2}x^2 - \frac{2x^{-2}}{-2} + c$

   $= \frac{2}{3}x^3 - \frac{5}{2}x^2 + \frac{1}{x^2} + c$

3.0 Basic Int

Complete the following table

Function	Integral (ignoring C)
0 $F(x) = \int 0 \, dx$	$F(x) = c$
1 $F(x) = \int 1 \, dx$	$F(x) = x + c$
x^n	
$\frac{1}{x}$	
e^{kx}	
$\cos kx$	
$\sin kx$ $F = \int \sin kx \, dx$	$F(x) = \frac{-\cos kx}{k}$
$\sec^2 kx$	$F(x) = \frac{\tan kx}{k}$
$\sec kx \times \tan kx$	$F(x) = \frac{\sec kx}{k}$
$-\csc kx \times \cot kx$	$F(x) = \frac{\csc kx}{k}$
$-\csc^2 kx$	$F(x) = \frac{\cot kx}{k}$

Examples: Use the integration techniques (sum and differences of functions) to evaluate:

1. Find the indefinite integral of $\int (5x^2 - 3)^2 dx$

$$F(x) = \int (25x^4 - 30x^2 + 9) \, dx$$

$$F(x) = \frac{25x^5}{5} - \frac{30x^3}{3} + \frac{9x^1}{1} + C$$

$$F(x) = 5x^5 - 10x^3 + 9x + C$$

2. Find the indefinite integral of $\int\left(x^{\frac{1}{3}}-4x+1\right)dx$

$F(x)=\dfrac{x^{4/3}}{4/3}-\dfrac{4x^2}{2}+\dfrac{x^1}{1}+c$

$=\frac{3}{4}x^{4/3}-2x^2+x+c$

3. Find the indefinite integral of $\int\left(\dfrac{1}{3x^4}-6x^4\right)^2 dx$

$=\left(\frac{1}{3x^4}-6x^4\right)\left(\frac{1}{3x^4}-6x^4\right)$

$=\frac{1}{9x^8}-2-2+36x^8$

$=\frac{1}{9}x^{-8}-4+36x^8$

$F(x)=\frac{1}{9}\left(\frac{x^{-7}}{-7}\right)-4\left(\frac{x^1}{1}\right)+36\left(\frac{x^9}{9}\right)+c$

$F(x)=-\dfrac{1}{63x^7}-4x+4x^9+c$

4. Find the indefinite integral of $\int\left(\dfrac{1}{\sqrt[3]{x}}+\sqrt{x}\right)dx$

5. Find the indefinite integral of $\int\left(\dfrac{t^2-\sqrt{t}-2}{t}\right)dt$

6. Find the indefinite integral of $\int\left(\sin x+\dfrac{2}{x}\right)dx$

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Integration by Substitution/Comparison

Outcomes: Solve integration questions using the substitution or comparison technique

Expand the integral then solve $\int 10x(5x^2-3)^2 dx$ $\quad 10x[25x^4 - 30x^2 + 9]$

$= \int (250x^5 - 300x^3 + 90x)dx$

$= 250\frac{x^6}{6} - 300\frac{x^4}{4} + 90\frac{x^2}{2}$

$= \frac{125x^6}{3} - 75x^4 + 45x^2$

Reverse Chain Rule (Substitution & Comparison Technique)

$\int 10x(5x^2-3)^2 dx \Rightarrow \int 10x[u]^2 \frac{du}{10x}$

$\frac{d}{dx}[u = 5x^2 - 3]$

$\frac{du}{dx} = 10x$

$\boxed{dx = \frac{du}{10x}}$

$= \int u^2 du$

$= \frac{u^3}{3}$

$= \frac{1}{3}(5x^2-3)^3$

$\int 10x(5x^2-3)^2 dx$

$= 10x\left[\frac{(5x^2-3)^3}{(3)\frac{d}{dx}(5x^2-3)}\right]$

$= \cancel{10x}\left[\frac{(5x^2-3)^3}{3(\cancel{10x})}\right]$

$= \frac{1}{3}(5x^2-3)^3$

Examples:

Find the Indefinite Integrals of the following.

a) $\int 2x(x^2-3)^7 dx$

b) $\int 28x^3(7x^4-3)^{11} dx$

c) $\int (x^2-5)^8 2x dx$

d) $\int \frac{x^2}{\sqrt{1-x^3}} dx$

e) $\int \frac{\ln x}{x} dx$

f) $\int (2+\sin x)^{10} \cos x dx$

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3.0 Basic Int

a) $\int 2x(x^2-3)^7\,dx$

$\frac{d}{dx}\left[u = x^2 - 3\right]$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$= \int 2x(u)^7 \frac{du}{2x}$

$= \int u^7\,du$

$= \frac{1}{8}u^8 + c$

$= \frac{1}{8}(x^2-3)^8 + c$

$\int 2x(x^2-3)^7\,dx$

$= [2x]\left[\frac{(x^2-3)^8}{8\ \frac{d}{dx}(x^2-3)}\right] + c$

$\ldots (2x)$

$= \frac{1}{8}(x^2-3)^8 + c$

b) $\int 28x^3(7x^4-3)^{11}\,dx$

$= (28x^3)\left[\frac{(7x^4-3)^{12}}{12(28x^3)}\right]$

$= \frac{1}{12}(7x^4-3)^{12} + c$

c) $\int (x^2-5)^8 \, 2x\,dx$

$\frac{d}{dx}(x^2-5) = 2x$

$$= [2x]\left[\frac{(x^2-5)^9}{9\,(2x)}\right] + C$$

$$= \frac{1}{9}(x^2-5)^9 + C$$

d) $\int \frac{x^2}{\sqrt{1-x^3}}dx$

$\frac{1}{(1-x^3)^{1/2}}$

$\frac{d}{dx}\left[u = 1-x^3\right]$

$\frac{du}{dx} = -3x^2$

cross multiply...

$dx = \frac{du}{-3x^2}$

$= \int x^2 (u)^{-1/2} \left[\frac{du}{-3x^2}\right]$

$= \int -\frac{1}{3}[u]^{-1/2}\,du$

$= -\frac{1}{3}\frac{[u]^{1/2}}{1/2} + c$

$= -\frac{2}{3}\sqrt{u} + c$

$= -\frac{2}{3}\sqrt{1-x^3} + c$

e) $\int \frac{\ln x}{x} dx$

$\frac{d}{dx}\left[u = \ln x\right]$

$\frac{du}{dx} = \frac{1}{x}$ cross multiply...

$dx = x\, du$

$= \int \frac{u}{x} \left[x\, du\right]$

$= \int u\, du$

$= \frac{u^2}{2} + C$

$= \frac{1}{2}(\ln x)^2 + C$

f) $\int (2+\sin x)^{10} \cos x dx$

$\frac{d}{dx}(2+\sin x) = \cos x$

$= \frac{(2+\sin x)^{11}}{11\,(\cos x)} [\cos x] + C$

$= \frac{1}{11}(2+\sin x)^{11} + C$