The Limits of Trigonometric Expressions

Outcomes: Find the limits of sine and cosine and simple modifications to them.

Warm up: Sketch the curve of sine   Sketch the curve of cosine

Investigate: Look at the left- and right-hand limits of $\sin\theta$ and $\cos\theta$ as $x \to 0$
Since we know that sine and cosine are continuous graphs state the value of
$$\lim_{\theta \to 0} \sin\theta \qquad \text{and} \qquad \lim_{\theta \to 0}\cos\theta.$$

Examples: Evaluate the following limits

a) $\displaystyle \lim_{\theta \to \pi} \frac{\sin\theta}{2}$

b) $\displaystyle \lim_{x \to 0}\left(\sin x + x\right).$

c) $\displaystyle \lim_{x \to \pi}\left(\sin x + \cos x\right)$

d) $\displaystyle \lim_{x \to \frac{\pi}{2}} \frac{\sin x + 1}{\cos x + 1}$

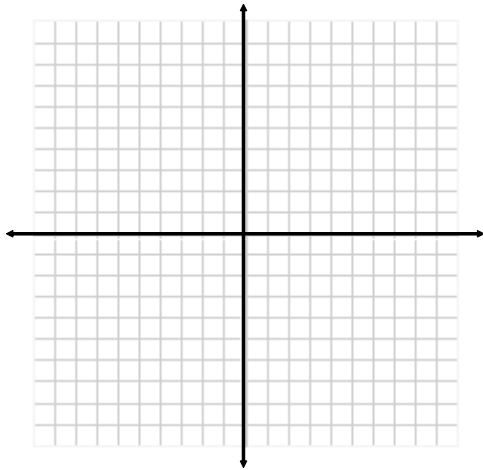e) $\displaystyle \lim_{x \to \frac{3\pi}{2}} \frac{\cos x}{2x}$

f) $\displaystyle \lim_{x \to 0} \frac{\cos 2x}{3\cos 3x}$

Investigate: Some limits which will be very important to trigonometric functions are:

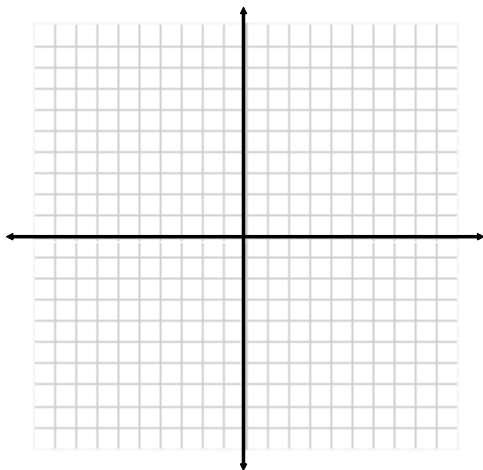$$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} \quad \text{or} \quad \lim_{\theta \to 0} \frac{\theta}{\sin \theta} \quad \text{and} \quad \lim_{\theta \to 0} \frac{\cos \theta - 1}{\theta}$$

These limits are necessary in order to find the derivatives of trigonometric functions.

Graph: $y = \dfrac{\sin \theta}{\theta}$ What does the graph indicate the $\lim\limits_{\theta \to 0} \dfrac{\sin \theta}{\theta}$ is equal to?

Graph: $y = \dfrac{\cos \theta - 1}{\theta}$ What does the graph indicate the $\lim\limits_{\theta \to 0} \dfrac{\cos \theta - 1}{\theta}$ is equal to?

Examples: Find the following limits

a) $\lim\limits_{\theta \to 0} \dfrac{\sin \theta}{2\theta}$

b) $\lim\limits_{\theta \to 0} \dfrac{\sin 3\theta}{\theta}$

c) $\lim\limits_{\theta \to 0} \dfrac{\sin^2 \theta}{\theta}$

d) $\lim\limits_{x \to 0} x \sec x$.

e) $\lim\limits_{x \to 0} \dfrac{\tan x}{\sin x}$.

f) $\lim\limits_{x \to 0} \dfrac{\sin 4x}{\sin 3x}$

g) $\displaystyle\lim_{\theta \to \frac{\pi}{2}} \frac{\sin \theta}{2\theta}$

h) $\displaystyle\lim_{x \to 0} \frac{\sin^2 3x}{\sin^2 4x}$

i) $\displaystyle\lim_{x \to \frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x}$

j) $\displaystyle\lim_{x \to 0} \frac{\tan x}{\tan 2x}$

Homework: Page 306: 1,2,7,9, 11,12,13,15,16,17,18,19,20,21,23,27,31,33