

CRITICAL NUMBERS

$$f'(x) = 0$$

$$f'(x) = \text{undefined}$$

$\frac{dy}{dx}$ is positive when function is increasing

$\frac{dy}{dx}$ is negative when function is decreasing

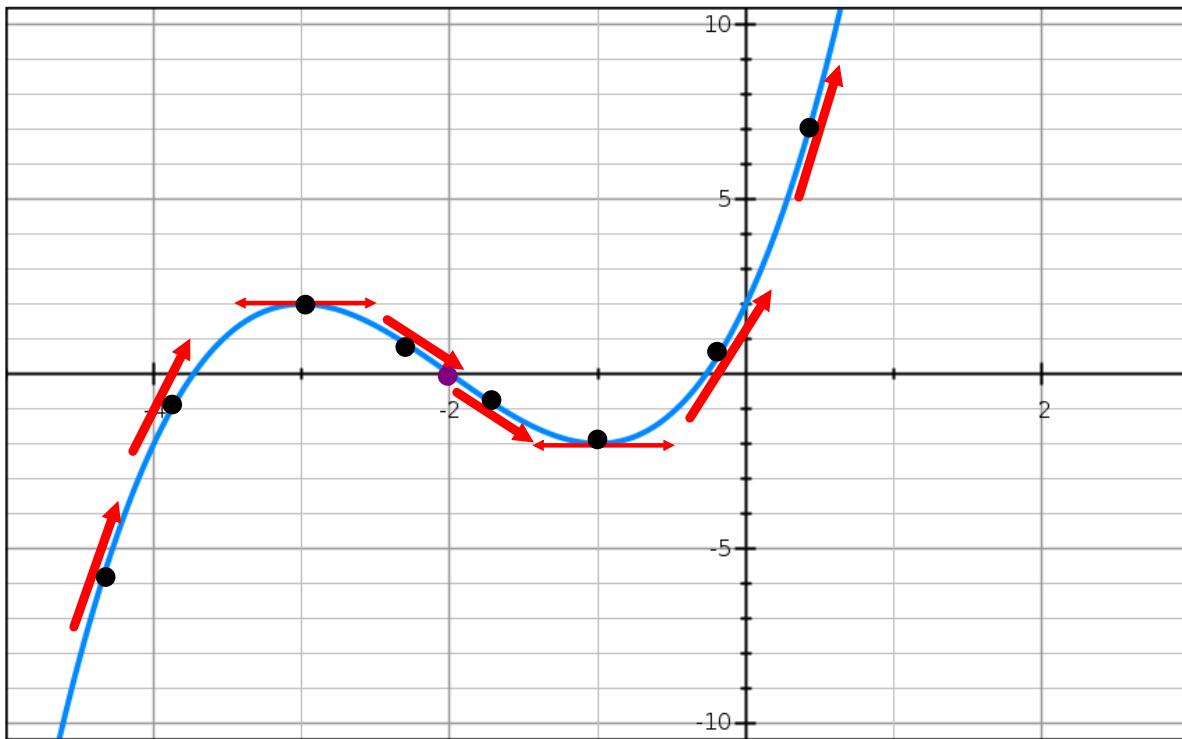
1. a) State the interval in which the following function is increasing and decreasing.

$$y = x^3 + 6x^2 + 9x + 2$$

- b) State the value where the first derivative is equal to zero.

- c) How can you use intervals of increase and decrease to determine if you have a max or min?

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CRITICAL NUMBERS: 2nd Derivative

$$f''(x) = 0$$

$$f''(x) = \text{undefined}$$

tangents lines above the curve

CONCAVE DOWN

$f''(x)$ is negative when the function is concave down

tangents lines below the curve

CONCAVE UP

$f''(x)$ is positive when the function is concave up

The Second Derivative Test

The point where a function changes from:
 Concave Up to Concave Down OR Concave Down to Concave Up is called a Point of Inflection.

Outcome: Identify the Maximum and Minimum of functions using the second derivative test.

1. State the critical values for the following functions and identify them as maximums, minimums or points of inflection.
2. Find the second derivative for each of the above functions. Determine the value of the 2nd derivative functions at the critical numbers for the first derivatives.

a) $y = x^3 + 6x^2 + 9x + 2$

$$\frac{dy}{dx} = 3x^2 + 12x + 9$$

$$0 = 3(x^2 + 4x + 3)$$

$$0 = 3(x+3)(x+1)$$

CN $x = -3$ $x = -1$ past chart

max	min
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$$\frac{d}{dx} \left[\frac{dy}{dx} = 3x^2 + 12x + 9 \right]$$

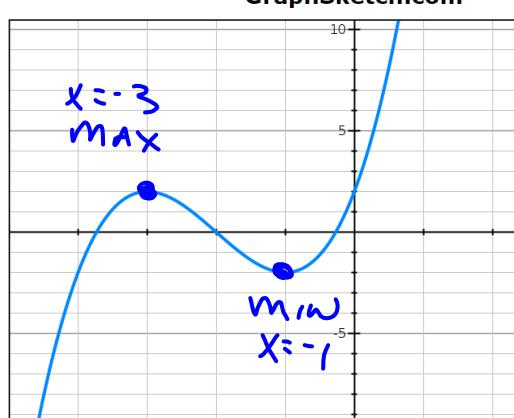
$$f''(x) = 6x + 12$$

$$f'(x) = 0 \quad CN = -3 \text{ and } -1$$

$$f''(-3) = 6(-3) + 12 = -6 \quad \text{concave down}$$

$$f''(-1) = 6(-1) + 12 = 6 \quad \text{concave up}$$

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b) $f(x) = x^3 - 3x + 1$

$$\begin{aligned}f'(x) &= 3x^2 - 3 \\0 &= 3(x^2 - 1) \\0 &= 3(x+1)(x-1)\end{aligned}$$

zero slopes

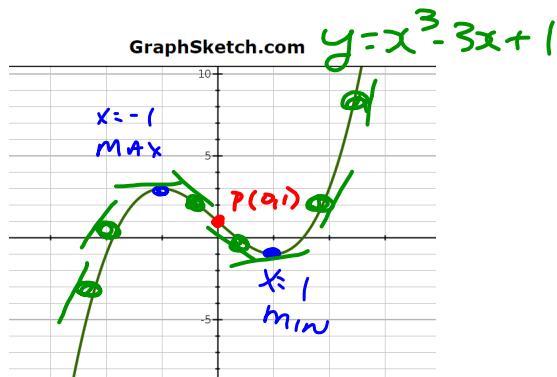
$$x = -1 \quad x = 1$$

$$\frac{d}{dx} [f'(x) = 3x^2 - 3]$$

$$f''(x) = 6x$$

zero slopes $f''(-1) = 6(-1) = -6$

$f''(1) = 6(1) = 6$



Somewhere $f''(x)$ changes from neg to pos?

$$f''(x) = 0 \quad \text{critical number } f''$$

$$f''(x) = 6x$$

$$6x = 0$$

$$x = 0$$

At $\boxed{x=0}$ not conc up or conc down
∴ point of inflection
 $f''(x) = 0$

$$f(x) = x^3 - 3x + 1$$

$$\begin{aligned}f(0) &= (0)^3 - 3(0) + 1 = 1 \\P(0, 1)\end{aligned}$$

c) $y = x^4 - 4x^3 - 8x^2 - 1$

$$\frac{dy}{dx} = 4x^3 - 12x^2 - 16x$$

ZERO SLOPE $O = 4x(x^2 - 3x - 4)$
 $O = 4x(x-4)(x+1)$
 $x=0 \quad x=4 \quad x=-1$



	$\frac{4x^3 - 12x^2 - 16x}{x+4}$			$f'(x)$	$f(x)$
	$x+4$	x	$x-1$		
$(-\infty, -1)$	-	-	-	NEG	DEC
$(-1, 0)$	+	-	-	POS	INC
$(0, 4)$	+	+	-	NEG	DEC
$(4, \infty)$	+	+	+	POS	INC



$$\frac{d}{dx} [f'(x) = 4x^3 - 12x^2 - 16x]$$

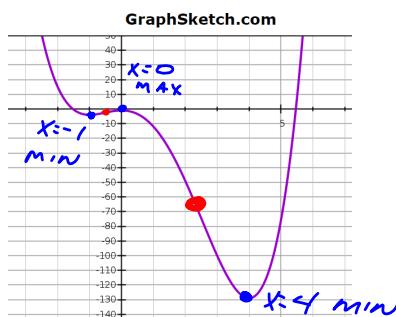
$$f''(x) = 12x^2 - 24x - 16$$

$$f''(-1) = 12(-1)^2 - 24(-1) - 16 = 20 \text{ concave up}$$

$$f''(0) = 12(0)^2 - 24(0) - 16 = -16 \text{ concave down}$$

$$f''(4) = 12(4)^2 - 24(4) - 16 = 80 \text{ concave up}$$

ZERO SLOPE



$$f''(x) = 12x^2 - 24x - 16$$

where $f''(x)=0$ are points of inflection:

$$0 = 12x^2 - 24x - 16$$

$$0 = 4(3x^2 - 6x - 4)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{6 \pm \sqrt{36 - 4(3)(-4)}}{2(3)}$$

$$x = \frac{6 \pm \sqrt{84}}{6} \quad \sqrt{4 \times 21}$$

$$x = \frac{6 \pm 2\sqrt{21}}{6}$$

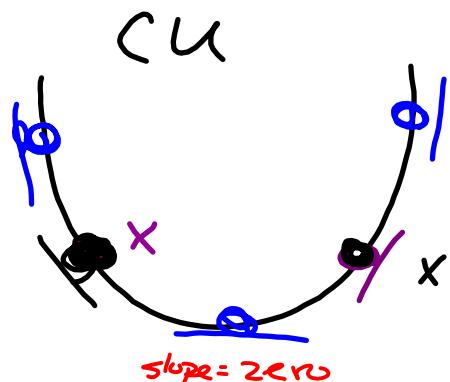
$$x = \frac{3 + \sqrt{21}}{3} \quad x = \frac{3 - \sqrt{21}}{3}$$

$$\text{d)} \quad y = x^3$$

How can we determine if a function has a local maximum or minimum using second derivatives?

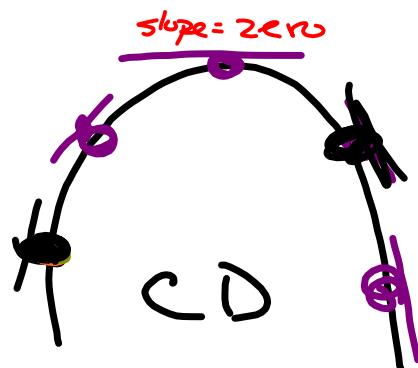
slope = zero

CONCAVITY $\begin{cases} \text{concave up IF } f''(x) \text{ is pos } f(x) \text{ is min} \\ \text{concave down IF } f''(x) \text{ is neg } f(x) \text{ is max} \end{cases}$



$f''(x) = \text{positive}$

$f''(x) = +$



$f''(x) = \text{negative}$

$f''(x) = \text{neg}$

2 thoughts ... $f'(x) = 0$

:

slope = zero

CONCAVE

Examples Find critical numbers of first and second derivatives. Find points of inflection, max and/or min values. Use this information to sketch the graphs.

a) $f(x) = x^2 + 2x - 3$

$$f'(x) = 2x + 2$$

$$0 = 2x + 2$$

$$x = -1 \quad \text{at } x = -1 \quad \text{slope} = \text{zero}$$

$$f''(x) = \frac{d}{dx}(2x + 2)$$

$$f''(x) = 2 \quad \dots \text{concave up for } x \in \mathbb{R}$$

never negative
"no zero", no P.I.

$\therefore f(-1)$ is slope zero and C.U



$f(-1)$ is a minimum

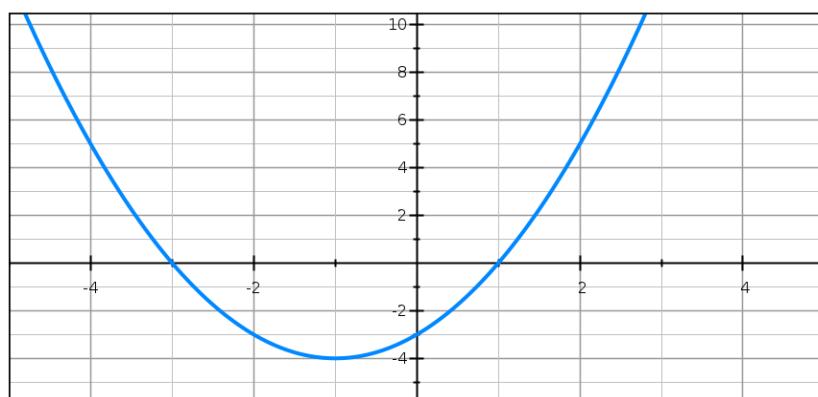
$$f(-1) = (-1)^2 + 2(-1) - 3$$

$$f(-1) = -4$$

$(-1, -4)$ is min.

-4 is min VALUE.

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$$\text{b) } f(x) = x^3 - 12x + 1$$

$$f'(x) = 3x^2 - 12$$

$$0 = 3x^2 - 12$$

$$0 = 3(x^2 - 4)$$

$$0 = 3(x+2)(x-2)$$

$$x = -2 \quad x = 2$$

$$f''(x) = 6x$$

$$0 = 6x$$

$$x = 0$$

PI $f(0)$

$$f(0) = (0)^3 - 12(0) + 1$$

$$f(0) = 1$$

$$P(0, 1)$$

zero slope + concave?

$$f''(-2) = 4(-2) = -8$$

CD \cap .. max $f(-2)$

$$f''(2) = 4(2) = 8$$

CU \cup .. min $f(2)$

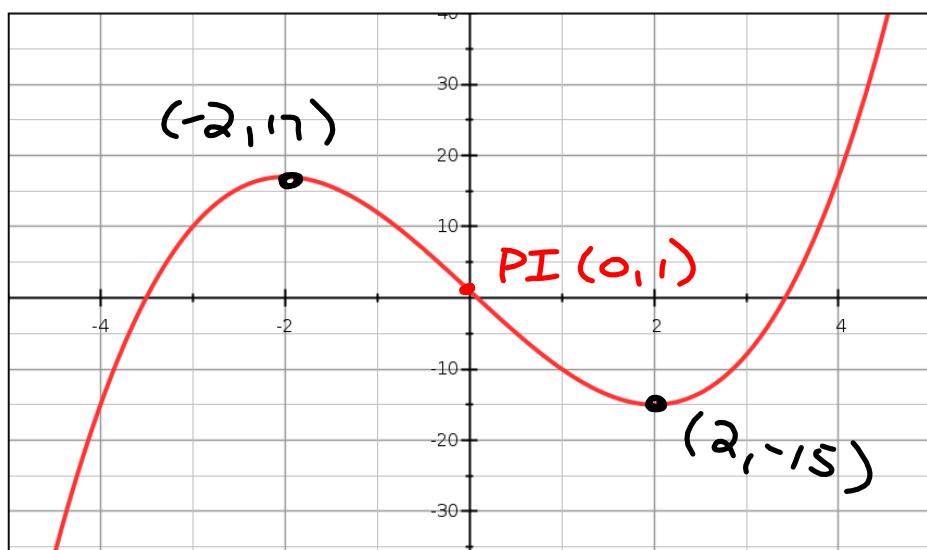
MAX
VALUE

$$f(-2) = (-2)^3 - 12(-2) + 1 = 17$$

MIN
VALUE

$$f(2) = (2)^3 - 12(2) + 1 = -15$$

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$$c) f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x + 1$$

$$f'(x) = x^2 - x - 6$$

$$0 = (x-3)(x+2)$$

$x=3$ $x=-2$] slopes = zero

$$f''(x) = \frac{d}{dx}[x^2 - x - 6]$$

$$f''(x) = 2x - 1$$

Point Inf $f''(x) = 0$

$$2x - 1 = 0$$

$$x = \frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = \frac{1}{3}\left(\frac{1}{2}\right)^3 - \frac{1}{2}\left(\frac{1}{2}\right)^2 - 6\left(\frac{1}{2}\right) + 1 \quad P.I.$$

$$f''(-2) = 2(-2) - 1 = -5 \text{ concave down}$$

$$f''(3) = 2(3) - 1 = 5 \text{ concave up}$$

MAX or MIN

Two Thoughts ... zero slope, concavity

$$f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x + 1$$

$f(-2)$ is max \cap

$$f(-2) = \frac{1}{3}(-2)^3 - \frac{1}{2}(-2)^2 - 6(-2) + 1$$

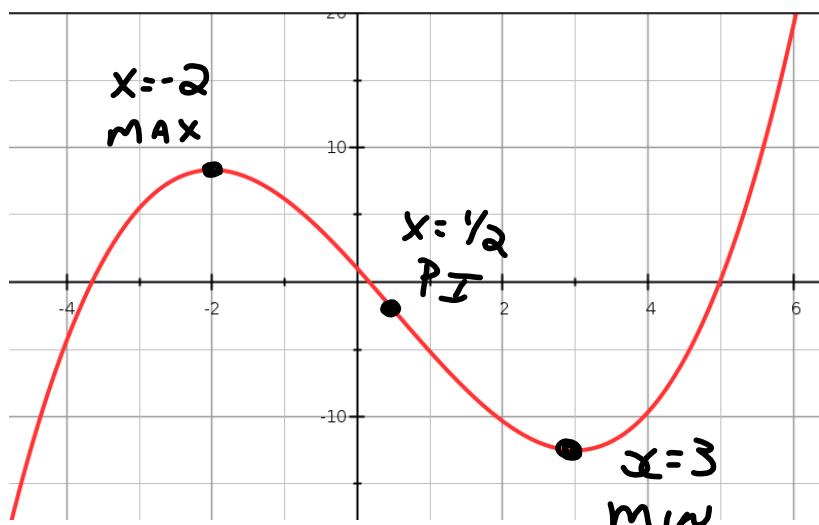
$$f(-2) =$$

$f(3)$ is min \cup

$$f(3) = \frac{1}{3}(3)^3 - \frac{1}{2}(3)^2 - 6(3) + 1$$

$$f(3) =$$

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