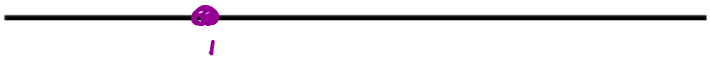
Applications of Derivatives – Velocity and Acceleration

- $displacement\ as\ a\ function\ of\ time = s(t)$
- $velocity\colon\ v(t) = \frac{d}{dt}(s) = \frac{ds}{dt} = s'(t)$
- $acceleration = \frac{d}{dt}(v) = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{d}{dt}(s)\right) = s''(t)$

1. The position in centimeters, relative to the origin, at any time t in seconds of an object moving along the x-axis is given by $s(t) = -2t^2 + 8t + 1 \quad t \geq 0$.



a) What is the position of the object at time $t = 4$?
b) What is the instantaneous velocity at $t = 4$?
c) Is the object moving away from or toward the origin (zero) at time $t = 4$? Justify.

a) $s(t) = -2t^2 + 8t + 1$

$s(4) = -2(4)^2 + 8(4) + 1$

$s(4) = 1$ cm

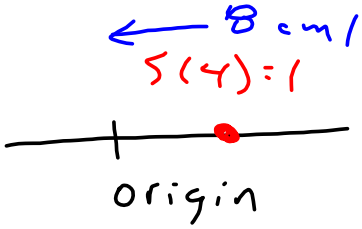
b) $s'(t) = -4t + 8$

$s'(4) = -4(4) + 8$

$= -8$ cm/s

OR 8 cm/s left

c)



From right of origin, moving left ∴ moving towards origin.

2. The position in metres, relative to the origin, at any time t in seconds of an object moving along the x-axis is given by: $s(t) = 3t - 2t^3$
   a) Determine the velocity at time of $t = 3$ s.
   b) Determine the acceleration at time of time of $t = 3$ s.
   c) Is the object increasing or decreasing its velocity at time of $t = 3$ s?

FORCES SAME DIRECTION ↓↓ OR ↑↑

forces opp direction ↑↓

a) $s(t) = 3t - 2t^3$

$v(t) = s'(t) = 3 - 6t^2$

$s'(3) = 3 - 6(3)^2$

$s'(3) = -51$ m/s

b) $a(t) = s''(t) = -12t$

$s''(3) = -12(3)$

$= -36 \text{ m/s}^2$

c) forces are in same direction ∴ velocity is increasing.

3. When will the velocity of a car traveling along a straight road be 100 km/h if its position in meters at time seconds is $s(t) = 5t + 2t^2$?

$$\frac{100\ \text{km}}{\text{h}} \cdot \frac{1\text{h}}{3600\text{s}} \cdot \frac{1000\text{m}}{1\text{km}}$$

$$v = 27.78\ \text{m/s}$$

$$s(t) = 5t + 2t^2$$

$$s'(t) = 5 + 4t$$

$$27.78 = 5 + 4t$$

$$t = 5.7\ \text{s}$$

4. A bicycle rider traveling along a straight road applies the brakes, and the rider's position in metres at any time t seconds is given by $s(t) = 3t - 0.75t^2$.
   a) What is the rider's initial velocity?
   b) How long does it take the rider to stop?

$\frac{d}{dt}[s(t) = 3t - 0.75t^2]$

$\frac{ds}{dt} = s'(t) = 3 - 1.5t$

a) $V(0) = 3 - 1.5(0)$

$V(0) = 3 \text{ m/s}$

b) $0 = 3 - 1.5t$

$t = 2 \text{ s}$

3.0 Velocity and Acc.2019

5. Find the acceleration at $t=1$, if time is measured in seconds and position in meters, for $s(t)=(4t^2+5)^3$

$$s(t)=(4t^2+5)^3$$

$$s'(t)=\frac{ds}{dt}=3(4t^2+5)^2(8t)$$

$$\frac{d}{dt}\left[v(t)=24t(4t^2+5)^2\right]$$

$$a(t)=v'(t)=s''(t)=\frac{ds}{dt^2}$$

$f=24t$ $\quad$ $g=(4t^2+5)^2$

$f'=24$ $\quad$ $g'=2(4t^2+5)(8t)$

$g'=16t(4t^2+5)$

$$a(t)=24(4t^2+5)^2+24t(16t(4t^2+5))$$

$$a(t)=24(4t^2+5)\left[(4t^2+5)+t(16t)\right]$$

$$a(t)=24(4t^2+5)(20t^2+5)$$

$$a(1)=24(4+5)(20+5)$$

$$a(1)=24(9)(25)$$

$$a(1)=5400$$

6. A snowmobiler traveling down a narrow bush road comes over a hill and sees another machine stalled 15 m directly ahead. The brakes are applied immediately and the snowmobiler's position is determined by $s(t)=12t-t^3,\ t\geq 0$ where t is in seconds and s(t) in meters. Will a crash occur? Justify.

$s(t) = 12t - t^3$

$s'(t) = 12 - 3t^2$

velocity = 0
find t
then s(t)

$0 = 12 - 3t^2$

$3t^2 = 12$

$t = 2$

$s(2) = 12(2) - (2)^3$

$s(2) = 16\ m$

Crash?

Yes, travel more than 15 m.