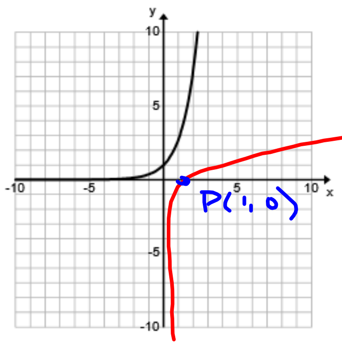
The Natural Logarithm

Objectives:

Use natural logarithms "$y = log_e x$ *is the same as* $y = lnx$" to simplify, change forms, solve equations.

Warm up: Given the function $y = e^x$ sketch $y = \ln x$



$y = log_e x$

$y = ln x$

$\ln x = \log_e x$ is called the natural logarithm.

1. Simplify the following:

$\log_e e^x$ a) $\ln e^x$ $= x$

b) $e^{\ln x}$ $= y \rightarrow x$ $\quad \log_e y = \ln x$

c) $\ln e$ $= 1$ $\quad \log_e e$

d) $\ln 1$ $= 0$ $\quad \ln e^0 = 0$

2. Solve for x in the following:

$\log_e x = 5$ a) $\ln x = 5$ $\quad x = e^5$

b) $[e^x = 20.086]$ ln $\quad x = 3$

c) $[e^{3-2x} = 4]$ ln

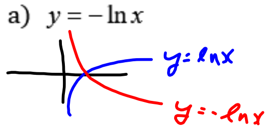
$3 - 2x = \ln 4$

$3 - \ln 4 = 2x$

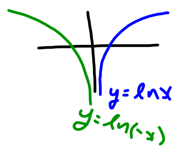
$x = \dfrac{3 - \ln 4}{2}$

3. Sketch the graphs of the following functions.

a) $y = -\ln x$



b) $y = \ln(-x)$



base = e

4. Express $\frac{2}{3}\ln x - 4\ln y + \ln(x+1)$ as a single logarithm.

$= \ln \dfrac{x^{2/3}(x+1)}{y^4}$

5. Find the domain of the function $f(x) = \ln(16 - x^2)$

$16 - x^2 = pos$ ... $16 - x^2 = 0$

$x = \pm 4$

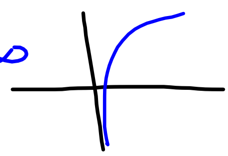
$D: (-4, 4)$

$y = \ln x$

$D: x > 0$

6. Find $\lim\limits_{x \to 4^-} \ln(16 - x^2)$

$= -\infty$ ... ln (small pos value) $= -\infty$



Homework: Page 375 #3,4,5,6,9(a,b),10.

The Derivative of Logarithmic Functions

Objectives: Find Derivatives of Logarithmic functions.

Use the exponential form and implicit differentiation to find the derivative of $y = \ln x$

"$y = \ln x$ is the same as $y = \log_e x$ is the same as $e^y = x$"

$\frac{d}{dx}\left[e^y = x\right]$

$\left[e^y\right]\frac{d}{dx}[y] = 1$

$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$

$\frac{d}{dx}\left[y = \ln x\right]$

$\frac{dy}{dx} = \frac{1}{x}$

SUMMARY: $y = \ln(u) \ldots \frac{dy}{dx} = \frac{1}{u} \cdot \frac{d}{dx}[u]$

Examples:

1. Differentiate

$f = x^2 \quad g = \ln x$

$f' = 2x \quad g' = \frac{1}{x}$

a) $y = x^2 \ln x$

$\frac{dy}{dx} = 2x\ln x + (x^2)(\frac{1}{x})$

$= 2x\ln x + x$

$= x\left[2\ln x + 1\right]$

b) $y = \ln(x^2 + 1)$

$\frac{dy}{dx} = \left[\frac{1}{x^2+1}\right]\frac{d}{dx}(x^2+1)$

$\frac{dy}{dx} = \frac{2x}{x^2+1}$

c) $y = (\ln x)^3$

$\frac{dy}{dx} = 3\left[\ln x\right]^2 \frac{d}{dx}\left[\ln x\right]$

$y' = \frac{3\left[\ln x\right]^2}{x}$

d) $y = x \ln x$

$f = x \quad g = \ln x$

$f' = 1 \quad g' = \frac{1}{x}$

$\frac{dy}{dx} = (1)(\ln x) + (x)(\frac{1}{x})$

$\frac{dy}{dx} = \ln x + 1$

4. Find $f'(x)$ if $f(x) = \log(x^2 + x)$

Homework: Page 383 #1(a,b,c,d,e,g,h,j,k,l) 3, 5(a,b,d)

e) $y=\ln\frac{x}{\sqrt{x+1}}$

$f=x$ $\quad g=(x+1)^{1/2}$

$f'=1$ $\quad g'=\frac{1}{2}(x+1)^{-1/2}[1]$

$$\frac{dy}{dx}=\frac{1}{\frac{x}{\sqrt{x+1}}}\frac{d}{dx}\left(\frac{x}{\sqrt{x+1}}\right)$$

$$\frac{dy}{dx}=\frac{(x+1)^{1/2}}{x}\left[\frac{(1)(x+1)^{1/2}-(x)(1/2(x+1)^{-1/2})}{[(x+1)^{1/2}]^2}\right]$$

$$=\frac{(x+1)^{1/2}}{x}\left[\frac{\frac{1}{2}(x+1)^{-1/2}[2(x+1)^1-x]}{(x+1)}\right]$$

$$=\frac{x+2}{2x(x+1)}$$

f) $y=\ln|x|$

$x>0$: $f(x)=\ln x$, $f'(x)=\frac{1}{x}$

$x<0$: $f(x)=\ln(-x)$, $f'(x)=\frac{1}{(-x)}(-1)$, $f'(x)=\frac{1}{x}$

$$\frac{d}{dx}\left[y=\ln|x|\right]$$

$$\frac{dy}{dx}=\frac{1}{x}$$

2. Find the derivative of $y=\log_3 x$

$$[3^y=x]\ln$$

$$\ln 3^y=\ln x$$

$$y\ln 3=\ln x$$

$$\left[y=\left(\frac{1}{\ln 3}\right)(\ln x)\right]\frac{d}{dx}$$

"constant"

$$\frac{dy}{dx}=\left(\frac{1}{\ln 3}\right)\left(\frac{1}{x}\right)$$

$y=5x^3$

$\frac{dy}{dx}=5(3x^2)$

$=15x^2$

$[y=\ln x]\frac{d}{dx}$

$\frac{dy}{dx}=\frac{1}{x}$

$$\frac{d}{dx}\left[y=\log_3 x\right]$$

$$\frac{dy}{dx}=\frac{1}{\ln 3\, x}$$

3. Develop a formula for finding $\frac{d}{dx}\log_b x$ using what you discovered above.

$$y=\log_b x$$

$$\frac{dy}{dx}=\frac{1}{\ln b\, x}$$

4. Find $f'(x)$ if $f(x) = \log(x^2 + x)$

$$f'(x) = \frac{1}{\ln 10\,(x^2+x)} \quad \frac{d}{dx}(x^2+x)$$

$$f'(x) = \frac{2x+1}{\ln 10\,(x^2+x)}$$

log "stretch"

$$\frac{1}{\log}$$