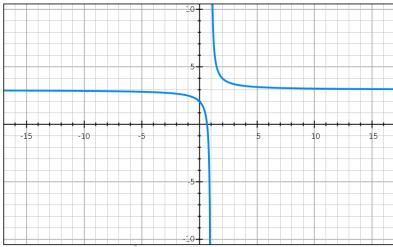


Asymptotes

Skills: Find limits at infinity. Find Domain of a function.
Outcomes: Find where vertical and horizontal asymptotes exist on a graph.

1. Given the function $f(x) = \frac{3x-2}{x-1}$
 - Sketch the function.
 - Write the equation for the horizontal asymptote of the function $f(x) = \frac{3x-2}{x-1}$
 - Write the equation for the vertical asymptote of the function $f(x) = \frac{3x-2}{x-1}$

GraphSketch.com



HORIZONTAL ASYMPTOTES

$$y = 3$$

$$\dots x \rightarrow \infty \quad y = 3$$

$$y = \lim_{x \rightarrow \infty} f(x)$$

$$y = \lim_{x \rightarrow \infty} \frac{3x-2}{x-1}$$

$$\frac{\frac{1}{\infty}}{\infty} = 0 \quad y = \text{const}$$

"largest degree"

$$\frac{1}{x^n}$$

$$y = \lim_{x \rightarrow \infty} \frac{(3x-2)/x}{(x-1)/x}$$

$$y = \lim_{x \rightarrow \infty} \frac{3 - \frac{2}{x}}{1 - \frac{1}{x}} = \frac{3-0}{1-0}$$

$y = 3$

vertical asymptote
* divide 2nd?

$$y = \frac{3x-2}{x-1}$$

$$x-1=0$$

$x=1$

2. Write the equation for the vertical and horizontal asymptotes of $y = \frac{x-3}{x+1}$

$$y = \lim_{x \rightarrow \infty} \frac{x-3}{x+1} \quad \frac{1/x}{1/x}$$

$$x+1=0$$

$$y = \lim_{x \rightarrow -\infty} \frac{1-3/x}{1+1/x}$$

$$x=-1$$

$$y=1$$

3. Write the equation for the vertical and horizontal asymptotes of $f(x) = \frac{x}{x^2-x-6}$

$$y = \lim_{x \rightarrow \infty} \frac{x(1/x^2)}{(x^2-x-6)(1/x^2)}$$

$$x^2-x-6=0$$

$$y = \lim_{x \rightarrow \infty} \frac{1/x}{1-1/x^2}$$

$$(x-3)(x+2)=0$$

$$y=0$$

$$x=3 \quad x=-2$$

4. Write the equation for the vertical and horizontal asymptotes of $f(x) = \frac{4x^2-x+2}{6x^2+5x+1}$

$$y = \lim_{x \rightarrow \infty} \frac{4-\frac{1}{x}+\frac{2}{x^2}}{6+\frac{5}{x}+\frac{1}{x^2}}$$

$$6x^2+5x+1=0$$

$$y = \frac{2}{3}$$

$$(2x+1)(3x+1)=0$$

$$x = -\frac{1}{2} \quad x = -\frac{1}{3}$$

5. Given: $f(x) = \frac{x^2 - 6x + 8}{x^2 - x}$

a) Write the equation for the vertical and horizontal asymptotes of $f(x) = \frac{x^2 - 6x + 8}{x^2 - x}$

b) Can a function cross a horizontal asymptote? Sketch $f(x) = \frac{x^2 - 6x + 8}{x^2 - x}$

$$y = \lim_{x \rightarrow \infty} \frac{1 - \frac{6}{x} + \frac{8}{x^2}}{1 - \frac{1}{x}} = 1$$

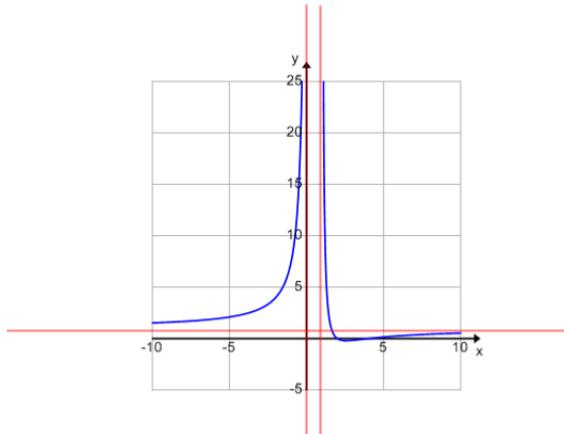
$$x^2 - x = 0$$

$$x(x - 1) = 0$$

$$y = 1$$

$$x = 0 \quad x = 1$$

$$y = \frac{x^2 - 6x + 8}{x^2 - x}$$



Yes, a function can cross an asymptote. Asymptotes represent the value the function is approaching as it nears infinity. If a function crosses its asymptote, you can use algebra to find the point of intersection. If a function does not cross its asymptote you can use algebra to show, there is no solution.

Example 5

$$1 = \frac{x^2 - 6x + 8}{x^2 - x}$$

$$x^2 - x = x^2 - 6x + 8$$

$$5x = 8$$

$$x = \frac{8}{5}$$

$$\text{Point } \left(\frac{8}{5}, 1\right)$$

Example 1

$$3 = \frac{3x - 2}{x - 1}$$

$$3x - 3 = 3x - 2$$

$$0 = 5 \dots \text{no solution}$$