Derivatives of the Primary Trigonometric Ratios

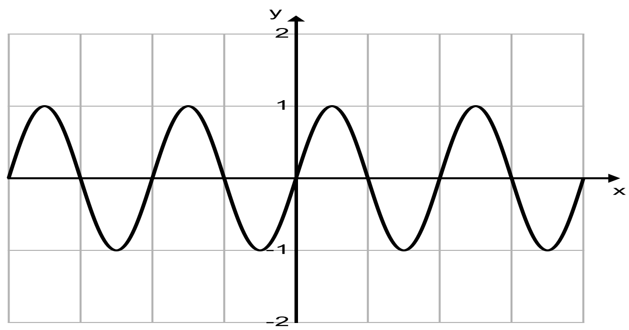
Given: $\lim_{x\to 0} \frac{\cos x - 1}{x} = 0$ and $\lim_{x\to 0} \frac{\sin x}{x} = 1$

Outcomes: Find derivatives of sine and cosine and simple modifications to them.

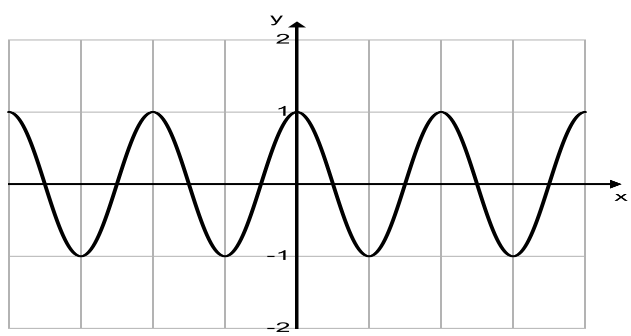
Recall: the derivative of a function was the slope of the tangent to a curve and that $\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$. This is the definition of derivative using the concept of first principles.

1. Determine the derivative of $y = \sin(x)$
   - Using first principles.

- Given the curve of $y = \sin(x)$. Use the concept of tangent line slopes to graphically find the derivative of $y = \sin(x)$.



2. Determine the derivative of $y = \cos(x)$.
   - Given the curve of $y = \cos(x)$. Use the concept of tangent line slopes to graphically find the derivative of $y = \cos(x)$.



If $y = \sin u$ or $y = \cos u$ is a composition of two functions, use the chain rule:

- $\frac{d}{dx}\sin u = \cos u \times \frac{du}{dx}$
- $\frac{d}{dx}\cos u = -\sin u \times \frac{du}{dx}$

Examples:

1. Find the derivative of the following

a) $y = \sin 2x$

b) $y = \sin(x^2 - 1)$

c) $y = x\sin x$

d) $y = \frac{x}{\sin x}$

e) $y = \cos^2 x$

f) $y = \cos(ax + b)$

g) $y = \cos(\sin x)$

h) Differentiate implicitly $\sin x + \sin y = 1$

2. Find the equation of the tangent line to $y = \frac{\sin x}{\cos 2x}$ at $x = \frac{\pi}{6}$

Homework:

Calculus, A First Course. McGraw-Hill Ryerson Limited, 1989.
Pg 313# 1 (second column), 2, 3(a,e), 4(a), 5(b), 11(a,c)

B 1. Find the derivative of y with respect to x in each of the following.

(a) $y = \cos(-4x)$

(b) $y = \sin(3x + 2\pi)$

(c) $y = 4\sin(-2x^2 - 3)$

(d) $y = -\frac{1}{2}\cos(4 + 2x)$

(e) $y = \sin x^2$

(f) $y = -\cos x^2$

(g) $y = \sin^{-2}(x^3)$

(h) $y = \cos(x^2 - 2)^2$

(i) $y = 3\sin^4(2 - x)^{-1}$

(j) $y = x\cos x$

(k) $y = \frac{x}{\sin x}$

(l) $y = \frac{\sin x}{1 + \cos x}$

(m) $y = (1 + \cos^2 x)^6$

(n) $y = \sin\frac{1}{x}$

(o) $y = \sin(\cos x)$

(p) $y = \cos^3(\sin x)$

(q) $y = x\cos\frac{1}{x}$

(r) $y = \frac{\sin^2 x}{\cos x}$

(s) $y = \frac{1 + \sin x}{1 - \sin 2x}$

(t) $y = \sin^3 x + \cos^3 x$

(u) $y = \cos^2\left(\frac{1 - \sqrt{x}}{1 + \sqrt{x}}\right)$

2. Find $\frac{dy}{dx}$ in each of the following.

(a) $\sin y = \cos 2x$

(b) $x \cos y = \sin(x + y)$

(c) $\sin y + y = \cos x + x$

(d) $\sin(\cos x) = \cos(\sin y)$

(e) $\sin x \cos y + \cos x \sin y = 1$

(f) $\sin x + \cos 2x = 2xy$

3. Find an equation of the tangent line to the given curve at the given point.

(a) $y = 2 \sin x$ at $\left(\frac{\pi}{6}, 1\right)$

(b) $y = \frac{\sin x}{\cos x}$ at $\left(\frac{\pi}{4}, 1\right)$

(c) $y = \frac{1}{\cos x} - 2 \cos x$ at $\left(\frac{\pi}{3}, 1\right)$

(d) $y = \frac{\cos^2 x}{\sin^2 x}$ at $\left(\frac{\pi}{4}, 1\right)$

(e) $y = \sin x + \cos 2x$ at $\left(\frac{\pi}{6}, 1\right)$

(f) $y = \cos(\cos x)$ at $x = \frac{\pi}{2}$

4. Find the critical numbers, the intervals of increase and decrease, and any maximum or minimum values.

(a) $y = \sin^2 x, \ -\pi \leqslant x \leqslant \pi$

(b) $y = \cos x - \sin x, \ -\pi \leqslant x \leqslant \pi$

5. Determine the concavity and find the points of inflection.

(a) $y = 2 \cos x + \sin 2x, \ 0 \leqslant x \leqslant 2\pi$

(b) $y = 4 \sin^2 x - 1, \ -\pi \leqslant x \leqslant \pi$

11. Find $\frac{dy}{dx}$ in each of the following.

(a) $y = \frac{1}{\sin(x - \sin x)}$

(b) $y = \sqrt{\sin\sqrt{x}}$

(c) $y = \sqrt[3]{x \cos x}$

(d) $y = \cos^3(\cos x) + \sin^2(\cos x)$

(e) $y = \sqrt{\cos(\sin^2 x)}$