5.0 Infinity

Going to Infinity

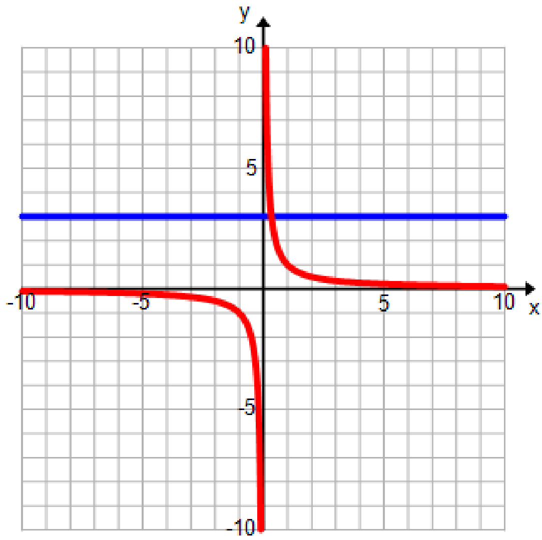
Warmup:

1. **Graph:** $f(x)=3$. $\boxed{y=3}$

   a) **Find** $\lim_{x\to 4} f(x) = 3$

   b) **Find** $\lim_{x\to 10} f(x) = 3$

   c) **Find** $\lim_{x\to \infty} f(x) = 3$



2. **Graph:** $f(x)=\frac{1}{x}$. $y=\frac{1}{x}$

   a) **Find** $\lim_{x\to 4} f(x) = \frac{1}{4}$

   b) **Find** $\lim_{x\to 10} f(x) = \frac{1}{10}$

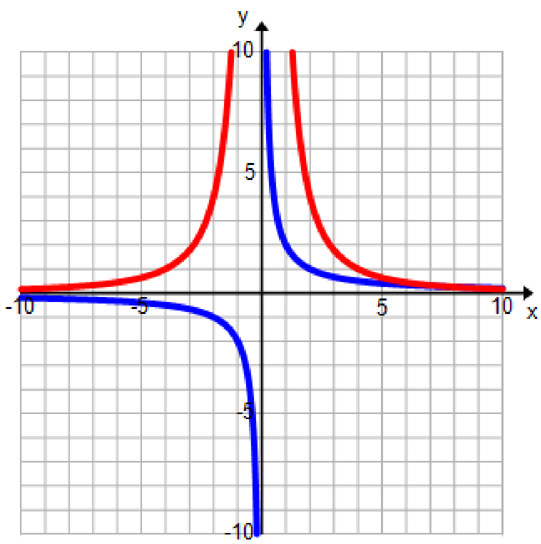
   c) **Find** $\lim_{x\to \infty} f(x) = 0$

3. **Graph:** $f(x)=\frac{2}{x}$.

   a) **Find** $\lim_{x\to 4} f(x) = \frac{2}{4} = \frac{1}{2}$

   b) **Find** $\lim_{x\to 10} f(x) = \frac{2}{10} = \frac{1}{5}$

   c) **Find** $\lim_{x\to \infty} f(x) = \frac{2}{\infty} = 2(0) = 0$



4. **Graph:** $f(x)=\frac{16}{x^2}$.

   a) **Find** $\lim_{x\to 4} f(x) = \frac{16}{4^2} = 1$

   b) **Find** $\lim_{x\to 10} f(x) = \frac{16}{10^2} = 0.16$

   c) **Find** $\lim_{x\to \infty} f(x) = \frac{16}{\infty^2} = (0)^2 = 0$

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We can conclude:

$$\lim_{x\to\infty}\left(\frac{1}{r}\right)^x = \frac{1}{r^x} = 0,\ if\ r>1 \quad \textbf{AND} \quad \lim_{x\to\infty} r^x = 0,\ if\ |r|<1$$

$$\frac{1}{\infty} = 0$$

Infinite Sequences

Definitions:

An infinite sequence is the range of a function which has the set of natural number as its domain. If the terms of an infinite sequence approach a unique finite value, that sequence is called a convergent sequence. A sequence which does not converge is called divergent.

OBJECTIVE – **find the value of an infinite convergent sequence.**

1. a) Determine the first five terms of the sequence defined by the function

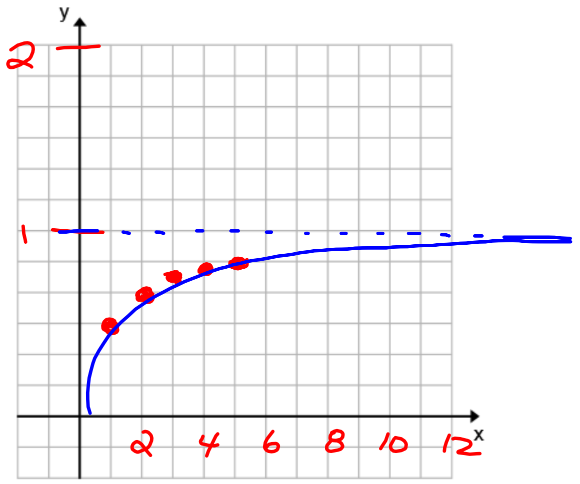
$$t(n) = \frac{n}{n+1} \quad \boxed{n \in N}$$

n	$\frac{n}{n+1}$
1	$\frac{1}{1+1} = \frac{1}{2}$
2	$\frac{2}{2+1} = \frac{2}{3}$
3	$\frac{3}{3+1} = \frac{3}{4}$
4	$\frac{4}{4+1} = \frac{4}{5}$
5	$\frac{5}{5+1} = \frac{5}{6}$

b) Plot the points of sequence.

"join for pattern"

10	$\frac{10}{10+1} = \frac{10}{11}$
1000	$\frac{1000}{1001}$



c) What do you think $\lim_{n\to\infty} f(n)$ is? What is the math that can justify this?

$$= \frac{\infty}{\infty+1} = 1$$

$$t_n = \frac{n}{n+1} \quad \left[\frac{\frac{1}{n}}{\frac{1}{n}}\right]$$

constants and/or rational functions?

$$t_n = \frac{1}{1+\frac{1}{n}}$$

$$\lim_{n\to\infty} \frac{1}{1+\frac{1}{n}} = \frac{1}{1+0} = \frac{1}{1} = 1$$

5.0 Infinity

As with functions, we can conclude:

$$\lim_{n\to\infty}\left(\frac{1}{r}\right)^n = \frac{1}{r^n} = 0, \text{ if } r > 1$$

"highest degree"
"make rational"

2. Find $\lim_{n\to\infty}\frac{2n-3}{n}$ $\frac{1/n}{1/n}$

$= \lim_{n\to\infty} \frac{2-\frac{3}{n}}{1} = \frac{2-0}{1} = 2$

3. Find $\lim_{n\to\infty}\frac{n^2-n}{2n^2+1}$ $\frac{\frac{1}{n^2}}{\frac{1}{n^2}}$

$= \lim_{n\to\infty} \frac{1-\frac{1}{n}}{2+\frac{1}{n^2}} = \frac{1-0}{2+0} = \frac{1}{2}$

4. Find the limit if they exist

a) $\lim_{n\to\infty}\frac{3n^2-5n+8}{2n^2+3n-7}$ $\frac{1/n^2}{1/n^2}$

$= \lim_{n\to\infty} \frac{3-\frac{5}{n}+\frac{8}{n^2}}{2+\frac{3}{n}-\frac{7}{n^2}}$

$= \frac{3-0+0}{2+0-0}$

$= \frac{3}{2}$

b) $\lim_{n\to\infty}(-1)^n$

n	$f(n)$
1	−1
2	1
3	−1
4	1
5	−1

not "approaching"

$\therefore$ no limit

c) $\lim_{n\to\infty}\frac{6n^3+1}{3n^4-n}$ $\frac{1/n^4}{1/n^4}$

$= \lim_{n\to\infty} \frac{\frac{6}{n}+\frac{1}{n^4}}{3-\frac{1}{n^3}}$

$= \frac{0+0}{3-0} = \frac{0}{3} = 0$

d) $\lim_{n\to\infty}\left(\frac{1}{2}\right)^n$

not rational function...

TABLE + pattern

n	$f(n)$
1	1/2
2	1/4
3	1/8
4	1/16
∞	0

Extra Practice: Page 50 #1 – 6, 8