

Limits at Infinity

Skill: Common factor polynomial expressions.

Outcome: Use limits at infinity to sketch curves that do not have horizontal asymptotes (polynomial functions).

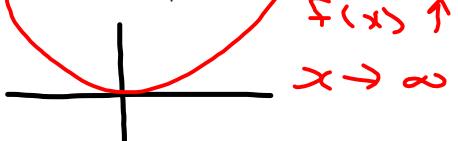
Warm up: Find the $\lim_{x \rightarrow \infty} x^2$

$$\lim_{x \rightarrow \infty} x^2 = (-\infty)^2$$

$$= \infty$$

$$= \infty$$

Investigate: Use the limits to help define the curve $y = x^2$



We can use intuition to recognize the product: infinity x infinity = infinity.

What about subtracting infinity: What is infinity cubed less infinity squared?

Skill – make sum/difference of infinity ‘products’ of infinity using factoring skills.

Examples:

Evaluate the following limits

$$\lim_{x \rightarrow \infty} (x^4 - x)$$

$$\lim_{x \rightarrow \infty} (x^3 - 2x + 1)$$

$$\lim_{x \rightarrow \infty} (x^4 - x^3)$$

$$\lim_{x \rightarrow \infty} x^4 \left(1 - \frac{1}{x^3}\right)$$

$$\lim_{x \rightarrow \infty} x^3 \left(1 - \frac{2}{x^2} + \frac{1}{x^4}\right)$$

$$\lim_{x \rightarrow \infty} x^5 \left(\frac{1}{x} - 1\right)$$

$$= (\infty)(\infty)(\infty)(\infty)(1-0) = (\infty)(\infty)(\infty)(1-0+0) = (\infty)^5(0-1) = -\infty$$

$$= \infty = \infty$$

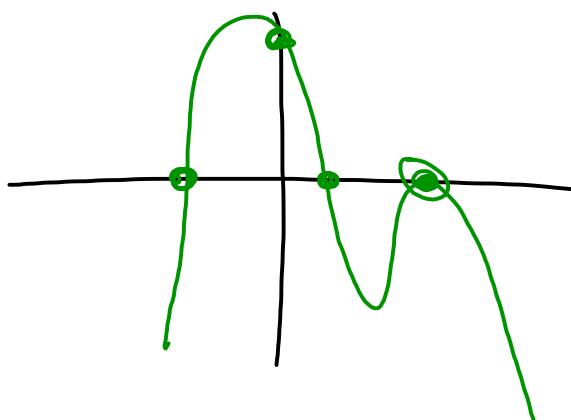
Sketch the graph of the curve $y = (x-3)^2(x+2)(1-x)$ by finding its intercepts and its limits.

$$\begin{aligned} & \lim_{x \rightarrow \infty} (x-3)^2(x+2)(1-x) \\ &= (\infty)^2(\infty)(-\infty) \\ &= -\infty \end{aligned}$$

$$\begin{aligned} & \lim_{x \rightarrow -\infty} (x-3)^2(x+2)(1-x) \\ &= (-\infty)^2(-\infty)(\infty) \\ &= -\infty \end{aligned}$$

$$\begin{aligned} & y\text{-int?} \\ & x=0 \\ & y = (0-3)^2(0+2)(1-0) \\ & y = 18 \end{aligned}$$

$$\begin{aligned} & x\text{-int} \\ & "2 \text{ zeros}" \\ & x-3=0 \quad x+2=0 \\ & x-3=0 \quad 1-x=0 \end{aligned}$$



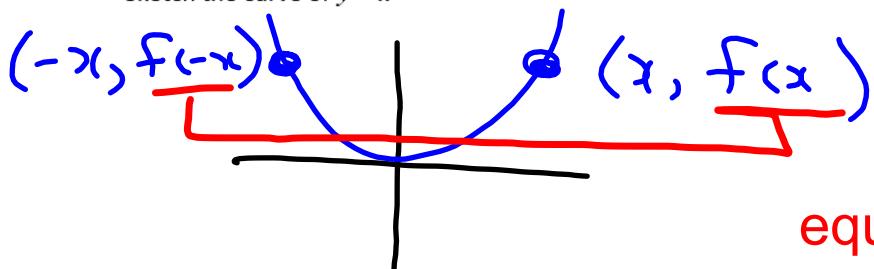
Odd and Even Functions

Even Functions are symmetric about the y-axis.

An even function is a function such that $f(x) = f(-x)$

An example of an even function is $y = x^2$

Sketch the curve of $y = x^2$

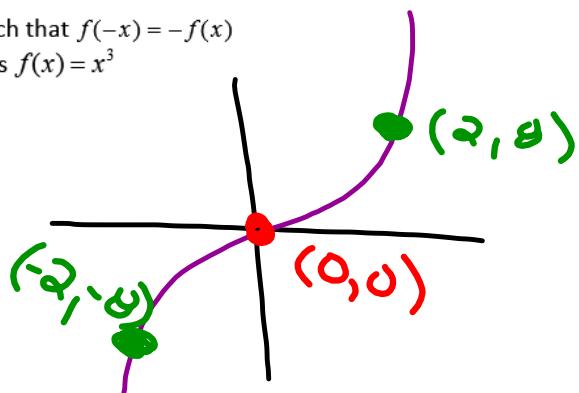


Odd functions are symmetric about the origin; x values are rotated 180° about the center to get the negative x-values.

An odd function is a function such that $f(-x) = -f(x)$

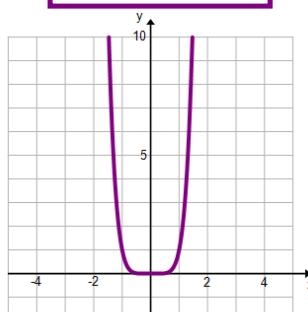
An example of an odd function is $f(x) = x^3$

Sketch the curve of $f(x) = x^3$

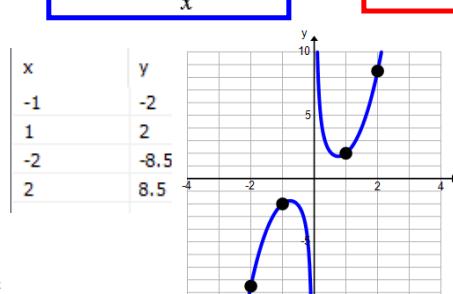


Examples: Justify algebraically why function is even or odd, or neither. Sketch the functions too.

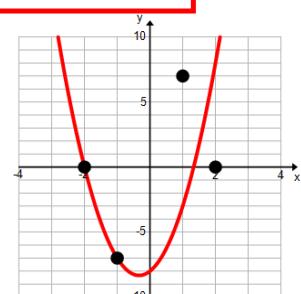
a) $f(x) = x^6$ is even.



b) $g(x) = x^3 + \frac{1}{x}$ is odd.



c) $h(x) = 3x^2 + 2x - 8$ is neither.



Homework: Page 206 #1,2 (Sketch the functions in 2)

- Odd/Even?
- Regions of increase/decrease.
- Regions of concavity.
- At infinity: asymptote or infinity?

$$y = x^3 + x$$

$$\frac{dy}{dx} = 0 \dots CN$$

$$\frac{d}{dx} \left(\frac{dy}{dx} \right) \dots CN$$

$$\frac{dy}{dx} = \text{zero (slope) } CN$$

$$3x^2 + 1 = 0 \quad \text{none}$$

	$3x^2 + 1$	$f'(x)$	$f(x)$
$(-\infty, \infty)$	+	pos	INC

" $f'(x)$ always increasing"

$$f''(x) = \frac{d}{dx} (3x^2 + 1)$$

$$f''(x) = 6x$$

$$0 = 6x$$

$$x = 0$$

	$6x$ on x	$f''(x)$	$f(x)$
$(-\infty, 0)$	-	NEG	CD
$(0, \infty)$	+	Pos	CU

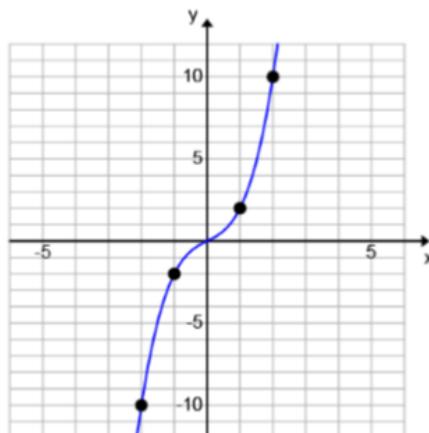
$$f(0) = (0)^3 + (0)$$

$$f(0) = 0$$

(matches odd too)

$f(0)$ is Point Inf

$$\begin{aligned} \lim_{x \rightarrow \infty} x^3 + x &= \lim_{x \rightarrow \infty} x^3 \left(1 + \frac{1}{x^2} \right) \\ &= (\infty)^3 (1+0) \\ &= \infty \end{aligned}$$



- Odd/Even?
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- Regions of concavity.
- At infinity: asymptote or infinity?

$$y = \frac{x^2}{x^2 - 16}$$

$$f'(x) = \frac{(2x)(x^2 - 16) - (x^2)(2x)}{(x^2 - 16)^2}$$

$$f'(x) = \frac{2x[x^2 - 16 - x^2]}{(x^2 - 16)^2}$$

$$f'(x) = \frac{-32x}{(x^2 - 16)^2}$$

$\frac{dy}{dx} = 0 \quad -32x = 0 \quad x = 0$
 $(x^2 - 16)^2 = 0 \quad x = \pm 4$

	$-32x$	$(x+4)^2(x-4)^2$	$f'(x)$	$f(x)$
$(-\infty, -4)$	+	+	pos	INC
$(-4, 0)$	+	+	pos	INC
$(0, 4)$	-	+	neg	DEC
$(4, \infty)$	-	+	neg	DEC


 local max
 $f(0)$

$$f''(x) = \frac{d}{dx} \left[\frac{-32x}{(x^2 - 16)^2} \right]$$

$$f''(x) = \frac{-32[(x^2 - 16)^2 - (-32x)][2(x^2 - 16)'(2x)]}{[(x^2 - 16)^2]^2}$$

$$f''(x) = \frac{-32(x^2 - 16)[(x^2 - 16) - 4x^2]}{(x^2 - 16)^4}$$

$$f''(x) = \frac{-32(-3x^2 - 16)}{(x^2 - 16)^3} \quad f''(x) = \frac{32(3x^2 + 16)}{(x^2 - 16)^3}$$

$f''(x) < zero$
 undefined

$$0 = \frac{32(3x^2 + 16)}{always +} \quad 0 = x^2 - 16 \quad x = \pm 4$$

$$f''(x) = \frac{32(3x^2 + 16)}{(x^2 - 16)^3}$$

outside the 4's, $f''(x)$ is positive... CU
 between the 4's, $f''(x)$ is negative... CD

