Find the local maximum and minimum values for:

$f(x) = \frac{\sqrt{3}}{2}x + cosx, [0, 2\pi]$.

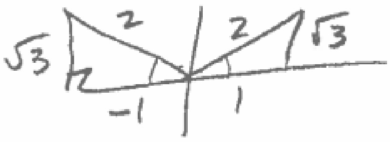
Justify using regions of increase and decrease or the second derivative test.

find slopes = zero $\quad f'(x) = 0$

$f'(x) = \frac{\sqrt{3}}{2} - \sin x$

$0 = \frac{\sqrt{3}}{2} - \sin x$

$\sin x = \frac{\sqrt{3}}{2} \quad [0, 2\pi]$




$x = \frac{\pi}{3} \quad x = \frac{2\pi}{3}$ ... slope = zero

$f''(x) = -\cos x$

$f''(\frac{\pi}{3}) = -\cos\frac{\pi}{3} = -\frac{1}{2}$ ∴ Concave Down, zero slope

MAX, $x = \frac{\pi}{3}$

$f''(\frac{2\pi}{3}) = -\cos\frac{2\pi}{3} = -(-\frac{1}{2}) = \frac{1}{2}$ ∴ Concave Up, zero slope

Min, $x = \frac{2\pi}{3}$

MAX $f(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}\left(\frac{\pi}{3}\right) + \cos\frac{\pi}{3} = \frac{\sqrt{3}\pi}{6} + \frac{1}{2} = \frac{\sqrt{3}\pi + 3}{6}$

min $f(\frac{2\pi}{3}) = \frac{\sqrt{3}}{2}\left(\frac{2\pi}{3}\right) + \cos\frac{2\pi}{3} = \frac{2\sqrt{3}\pi}{6} - \frac{1}{2} = \frac{2\sqrt{3}\pi - 3}{6}$

Find the local maximum and minimum values for:

$f(x) = \frac{\sqrt{3}}{2}x + cosx, [0,2\pi]$.

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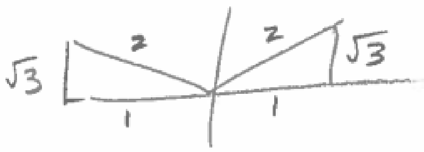



$\sin x = \frac{\sqrt{3}}{2}$

$[0, 2\pi]$

$x = \frac{\pi}{3}$ $\quad x = \frac{2\pi}{3}$

| | $\frac{\sqrt{3}}{2} - \sin x$ | $f'(x)$ | $f(x)$ |
|---|---|---|---|
| $(0, \frac{\pi}{3})$ | + | + | INC |
| $(\frac{\pi}{3}, \frac{2\pi}{3})$ | − | − | DEC |
| $(\frac{2\pi}{3}, 2\pi)$ | + | + | INC |

> max $f(\frac{\pi}{3})$

> min $f(\frac{2\pi}{3})$

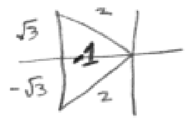
MAX $f(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}\left(\frac{\pi}{3}\right) + \cos\frac{\pi}{3} = \frac{\sqrt{3}\pi}{6} + \frac{1}{2} = \frac{\sqrt{3}\pi + 3}{6}$

min $f(\frac{2\pi}{3}) = \frac{\sqrt{3}}{2}\left(\frac{2\pi}{3}\right) + \cos\frac{2\pi}{3} = \frac{2\sqrt{3}\pi}{6} - \frac{1}{2} = \frac{2\sqrt{3}\pi - 3}{6}$

Find the local maximum and minimum for $f(x) = x + 2\sin x, \quad [0, 2\pi]$ Justify.

$$\frac{dy}{dx} = 0 \quad \text{OR} \quad u$$

$$x = \qquad\qquad x =$$




5. Find the local maximum and minimum for $f(x) = x + 2\sin x, \quad [0, 2\pi]$ Justify.

[3]

$f'(x) = 1 + 2\cos x$

$0 = 1 + 2\cos x$

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3} \qquad x = \frac{4\pi}{3}$

$f''(x) = -2\sin x$

$f''\left(\frac{2\pi}{3}\right) = -2\sin\left(\frac{2\pi}{3}\right) = \text{neg} \therefore \text{CD, max}$

$f''\left(\frac{4\pi}{3}\right) = -2\sin\left(\frac{4\pi}{3}\right) = \text{pos} \therefore \text{CU, min}$

| | $1+2\cos x$ | $f'(x)$ | $f(x)$ |
|---|---|---|---|
| $\left[0, \frac{2\pi}{3}\right)$ | pos | pos | inc |
| $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$ | neg | neg | dec |
| $\left(\frac{4\pi}{3}, 2\pi\right]$ | pos | pos | inc |

local max

local min

$f\left(\frac{2\pi}{3}\right) = \frac{2\pi}{3} + 2\left(\frac{\sqrt{3}}{2}\right) = \frac{2\pi}{3} + \sqrt{3}$ is "max"

$f\left(\frac{4\pi}{3}\right) = \frac{4\pi}{3} + 2\left(\frac{-\sqrt{3}}{2}\right) = \frac{4\pi}{3} - \sqrt{3}$ is "min"