MATH 31 18. Name Key

Applications of Derivatives – Related Motion Date ______

4

1. Sand pouring from a conveyor belt forms a conical pile, the radius of which is $\frac{3}{4}$ of the height. The sand is piling up at a constant rate of $0.5\,{}^{m^3}\!/_{\min}$. At what rate is the height of the pile growing 3 minutes after the pouring starts?

$\frac{dV}{dt} = 0.5\ m^3/min$

$\frac{dh}{dt} = ?$

$V = \frac{1}{3}\pi r^2 h$ and $r = \frac{3}{4}h$

$V = \frac{1}{3}\pi\left(\frac{3}{4}h\right)^2(h)$

$V = \frac{3\pi}{16}h^3$ 1

"Rates"

$\frac{dV}{dt} = \frac{9\pi}{16}h^2\frac{dh}{dt}$ ... h when $t = 3$ minutes

$V = 3 \times 0.5\ m^3/min$

$V = 1.5\ m^3$

$1.5 = \frac{3\pi}{16}h^3$

$h^3 = \frac{8}{\pi}$ 1

$h = \frac{2}{\sqrt[3]{\pi}}$

$0.5 = \frac{9\pi}{16}\left(\frac{2}{\sqrt[3]{\pi}}\right)^2\frac{dh}{dt}$

1 $0.5 = \frac{9\pi}{16}\left(\frac{4}{\pi^{2/3}}\right)\frac{dh}{dt}$ ... $\frac{9\pi^{1/3}}{4}\frac{dh}{dt}$

$\frac{2}{9\pi^{1/3}} = \frac{dh}{dt} = 0.15\ m/min$ 1

5

2. When a small pebble is dropped in a pool of still water, it produces a circular wave that travels outward at a constant speed of 15 cm/s. At what rate is the area inside the wave increasing when

a) the area is $400\pi\ cm^2$?

$A = \pi r^2$ ... $400\pi = \pi r^2$

$400 = r^2$

$r = 20$ 1

1 $\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(20)(15)$

$= 600\pi\ cm^2/s$ 1

b) 4 s have elapsed?

$A = \pi r^2$ ... $r = 4(15)$

$r = 60\,cm$ 1

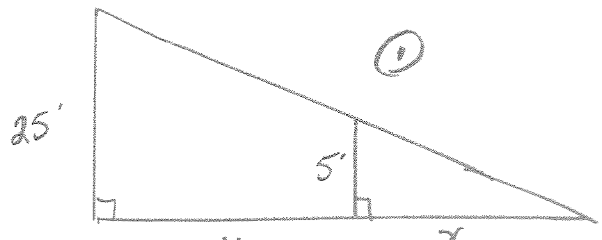
$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(60)(15)$

$= 1800\pi\ cm^2/s$ 1

4

3. A lady, 5 feet tall, is walking away from a lamppost that is 25 feet high. The lady is 15 feet from the lamppost and walking at a rate of 2.5 feet/second. At what rate is her shadow increasing? [Round your answer to the nearest tenth.]



1

$\frac{dy}{dt} = 2.5$ ft/s

$\frac{\text{small} \Delta}{\text{BIG} \Delta}$ $\frac{5}{25} = \frac{x}{x+y}$ OR $\frac{5}{x} = \frac{25}{x+y}$ etc.

$5x + 5y = 25x$ 1

$5y = 20x$

$y = 4x$

$\frac{dy}{dt} = 4\frac{dx}{dt}$ 1

$2.5 = 4\frac{dx}{dt}$

$\frac{dx}{dt} = \frac{5}{8}$ OR $\boxed{0.6 \text{ ft/s}}$ 1

$= 0.625$

15

4. Gas is escaping from a spherical weather balloon at a rate of $50 \, ^{cm^3}/_{\min}$. How fast is the surface area, S, shrinking when the radius is 15 meters? $V = \frac{4}{3}\pi r^3$ and $SA = 4\pi r^2$

$\frac{dV}{dt} = -\frac{50 cm^3}{min}$

$r = 1500 cm$

$A = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$

1 $\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$ ... $r = 1500$

1 $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dr}{dt} = ?$

$-50 = 4\pi (1500)^2 \frac{dr}{dt}$

1 $\frac{dr}{dt} = \frac{-50}{4\pi (1500)^2}$

1 $\frac{dA}{dt} = 8\pi(1500)\left(\frac{-50}{4\pi(1500)^2}\right)$

$\frac{dA}{dt} = \frac{-100}{1500} = -\frac{1}{15}$ cm²/min

1

OR $\frac{dA}{dt} = -0.067$ cm²/min