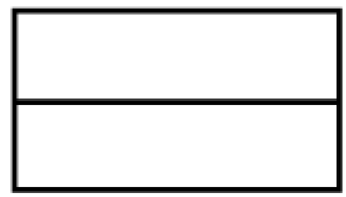
Math 31 **Name: ______________________**

Applications of Derivatives and Curve Sketching Exam - 2017

A. Problem Solving: Use derivatives to solve problems. Use exact values (unless otherwise directed) and proper rates when answering your questions. 4 marks each.

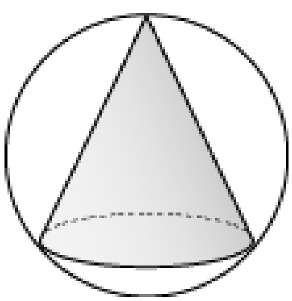
1. A rancher wants to fence an area of 3456 m^2 into a rectangular field and divide it in half with a fence parallel to one of the sides of the rectangle. The rancher wants to minimize the amount of fence required for this area. Determine the dimensions of the field to minimize the amount of fence material.

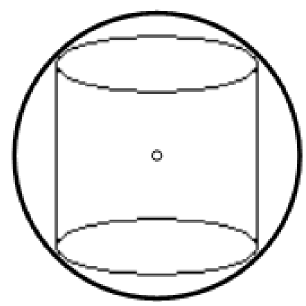


2. You have a sphere with a radius of 10.0 cm. Determine the exact radius and height of the largest cylinder **<u>or</u>** the largest right circular cone that can be inscribed in this sphere. $V_{sphere} = \frac{4}{3}\pi r^3$

$V_{cone} = \frac{1}{3}\pi r^2 h$

$V_{cylinder} = \pi r^2 h$



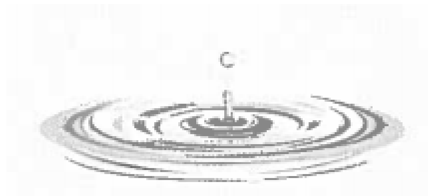


3. Solve one of the following problems:

   A. A straight level road crosses a railroad track at a right angle. A car is on the road 2.0 km from the crossing, traveling at 95km/h toward the crossing. At the same time, a train is 1.5 km from the crossing and traveling 75 km/h toward the crossing. For this moment in time, determine the rate of change in the distance between the car and the train.

   B. A small stone is dropped in a pond creating a circle with a radius change of 4.0 cm/s.

      i. Determine the exact rate of change to the inside area of the circle with respect to radius when the area of the circle in the water is $625\pi\ cm^2$.
      ii. Determine the exact rate of change to the inside area of the circle with respect to time 5.0 seconds after the stone hits the water.



$A = \pi r^2$

4. A conical paper cup 8.0 cm across the top and 12.0 cm deep is being filled with water at a rate of 2.5 cm^3/s. How fast is the level of water rising 20.0 seconds after the filling begins? [This answer can be rounded to nearest hundredth.] $V_{cone} = \frac{1}{3}\pi r^2 h$

Section B – Curve Sketching. In order to receive full marks for the questions, you must show all pertinent work.

1. Identify the intervals of increase and decrease for $y = f(x)$ given:

$$f'(x) = \frac{x^2 - 25}{x^2 - 6x + 9}$$

[3]

2. For the curve $= 3x^4 - 4x^3 - 54x^2 + 108x$, find the local maximum and/or minimum values using the first and/or second derivative rules.

[3]

3. For the curve $f(x) = 3x^4 - 6x^3 + 5$

   a) Find the intervals of concavity.

   b) Find any inflection points.

[3]

4. Given, $f(x) = \frac{6x^2+5x+1}{12-3x^2}$

   Find the vertical and horizontal asymptotes of the curve.

[2]

5. Sketch the function given:

[3] The domain is $\{x \neq \pm 2\}$; the vertical asymptotes are $x = 2$ and $x = -2$. The intercepts are both at 0 and $f(0)$ is a local minimum. The horizontal asymptote is the line $y = -4$. The interval of decrease is $(-\infty, -2) \cup (-2, 0)$. The interval of increase is $(0, 2) \cup (2, \infty)$. The function is concave upward on $(-2, 2)$. The function is concave downward on $(-\infty, -2) \cup (2, \infty)$. There are no points of inflection.