MATH 31 NAME KEY

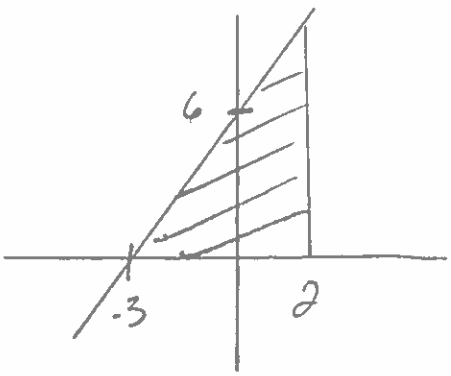
/18

AREA ASSIGNMENT

3 each

1. Find the area of the region under

   a) $y = 2x + 6$ from -3 to 2



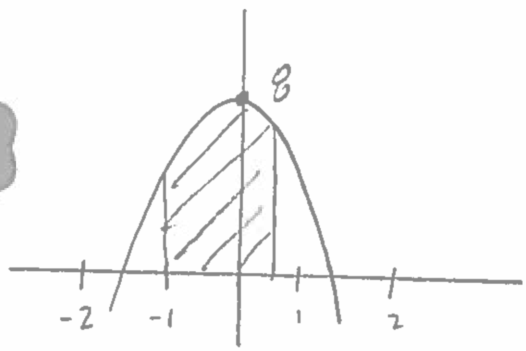
$$A = \int_{-3}^{2} 2x+6\,dx = \left[x^2+6x\right]_{-3}^{2}$$

$$= \left[(2)^2+6(2)\right] - \left[(-3)^2+6(-3)\right]$$

$$= 4 + 12 - 9 + 18$$

$$= 25$$

   b) $y = 8 - 4x^2$ from -1 to $^1/_2$



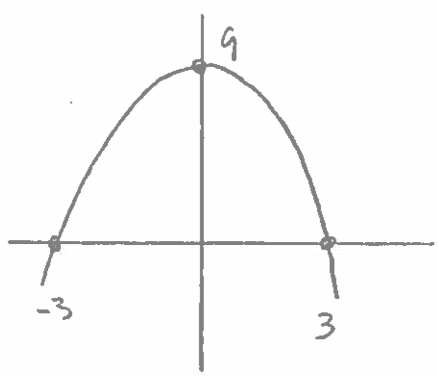
$$A = \int_{-1}^{1/2} 8-4x^2\,dx = \left[8x - \frac{4}{3}x^3\right]_{-1}^{1/2}$$

$$= \left[8(1/2) - \frac{4}{3}(1/2)^3\right] - \left[8(-1) - \frac{4}{3}(-1)^3\right]$$

$$= 4 - \frac{1}{6} + 8 - \frac{4}{3}$$

$$= \frac{21}{2}$$

2. Find the area bounded by the curve $y = 9 - x^2$ and the x-axis.



$$A = \int_{-3}^{3} 9-x^2\,dx = \left[9x - \frac{1}{3}x^3\right]_{-3}^{3}$$

$$= \left[9(3) - \frac{1}{3}(3)^3\right] - \left[9(-3) - \frac{1}{3}(-3)^3\right]$$

$$= 27 - 9 + 27 - 9$$

$$= 36$$

3. Find the area between the given curves:

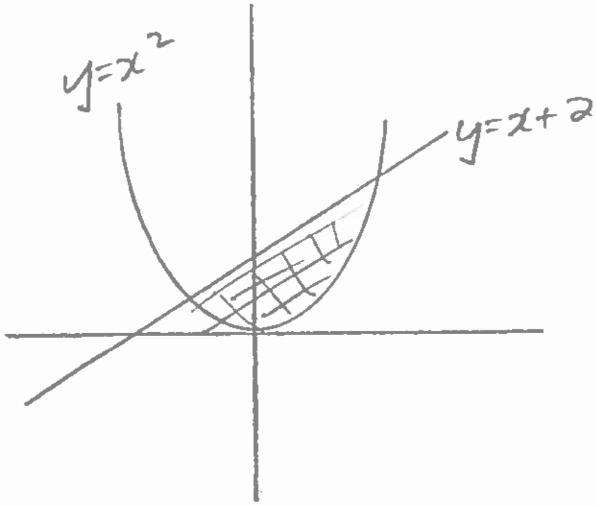
a) $y = x + 2$ *and* $y = x^2$

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, \quad x = -1$



$$A = \int_{-1}^{2} x + 2 - x^2 \, dx$$

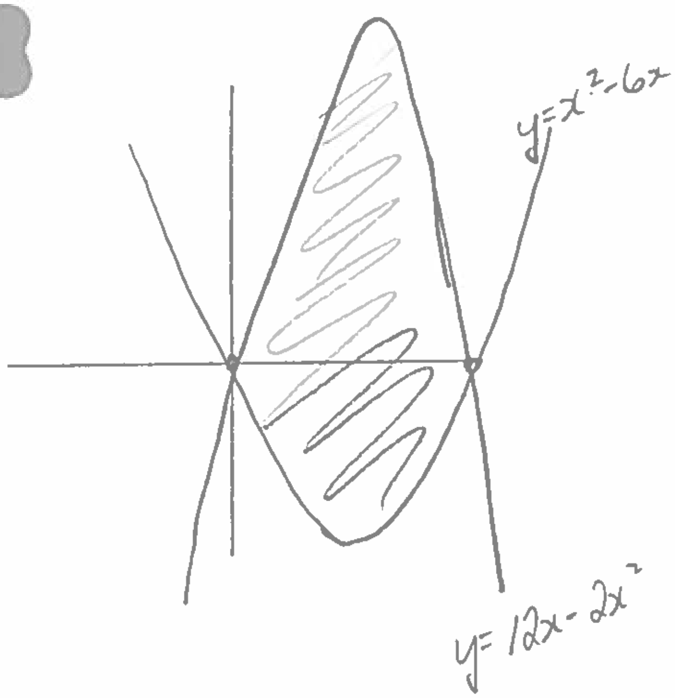
$$= \left[-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x \right]_{-1}^{2}$$

$$= \left[-\frac{1}{3}(2)^3 + \frac{1}{2}(2)^2 + 2(2) \right] - \left[-\frac{1}{3}(-1)^3 + \frac{1}{2}(-1)^2 + 2(-1) \right]$$

$$= -\frac{8}{3} + 2 + 4 - \frac{1}{3} - \frac{1}{2} + 2$$

$$= \frac{9}{2}$$

b) $y = x^2 - 6x$ *and* $y = 12x - 2x^2$



$x^2 - 6x = 12x - 2x^2$

$3x^2 - 18x = 0$

$3x(x-6) = 0$

$x = 0 \quad x = 6$

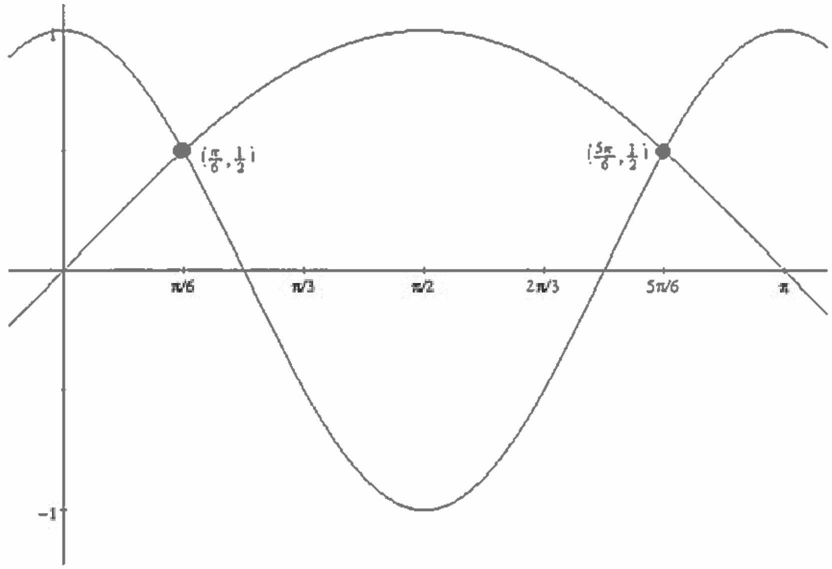
$$A = \int_0^6 (12x - 2x^2) - (x^2 - 6x) \, dx$$

$$A = \int_0^6 18x - 3x^2 \, dx$$

$$A = \left[9x^2 - x^3 \right]_0^6 = 9(6)^2 - (6)^3$$

$$= 108$$

4. Find the area between the given curves, $y = \sin x$ and $y = \cos 2x$, from $x = 0$ to $x = \pi$..
Shade the sketch to show your regions.



$$A = \int_0^{\pi/6} \cos(2x) - \sin(x) + \int_{\frac{\pi}{6}}^{5\pi/6} \sin(x) - \cos(2x) + \int_{\frac{5\pi}{6}}^{\pi} \cos(2x) - \sin(x)$$

$$= \left[\frac{1}{2}\sin(2x) + \cos x\right]_0^{\frac{\pi}{6}} + \left[-\cos(x) - \frac{1}{2}\sin(2x)\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} + \left[\frac{1}{2}\sin(2x) + \cos(x)\right]_{\frac{5\pi}{6}}^{\pi}$$

"A_1" "B" "C"

$$A_1 = \left(\frac{1}{2}\sin\frac{\pi}{3} + \cos\frac{\pi}{6}\right) - \left(\frac{1}{2}\sin(0) + \cos(0)\right) = \left(\frac{1}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}(0) + 1\right)$$

$$= \frac{\sqrt{3}}{4} + \frac{2\sqrt{3}}{4} - \frac{4}{4} = \frac{3\sqrt{3} - 4}{4}$$

$$B. \left(-\cos\frac{5\pi}{6} - \frac{1}{2}\sin\frac{5\pi}{3}\right) - \left(-\cos\frac{\pi}{6} - \frac{1}{2}\sin\frac{\pi}{3}\right) = -\left(-\frac{\sqrt{3}}{2}\right) - \frac{1}{2}\left(-\frac{\sqrt{3}}{2}\right) - \left(-\frac{\sqrt{3}}{2} - \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right.\right.$$

$$= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4}$$

$$= \frac{6\sqrt{3}}{4} = \frac{3\sqrt{3}}{2}$$

$$\text{"}C\text{"} = \left(\frac{1}{2}\sin(2\pi) + \cos(\pi)\right) - \left(\frac{1}{2}\sin\frac{5\pi}{3} + \cos\frac{5\pi}{6}\right)$$

$$= \left[\frac{1}{2}(0) + (-1)\right] - \left[\frac{1}{2}\left(-\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{3}}{2}\right]$$

$$= -1 + \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2}$$

$$= \frac{-4 + \sqrt{3} + 2\sqrt{3}}{4}$$

$$= \frac{3\sqrt{3} - 4}{4}$$

$$A_{total} = \frac{3\sqrt{3} - 4}{4} + \frac{6\sqrt{3}}{4} + \frac{3\sqrt{3} - 4}{4}$$

$$= \frac{12\sqrt{3} - 8}{4}$$

$$= \boxed{3\sqrt{3} - 2} \qquad \approx 3.196$$