MATH 31 Name ______________

DERIVATIVES $\frac{}{33}$ Date ______________

1. Given $f(x) = x^2 + 4x - 5$.
   a) Find the derivative for $f(x)$ using **first principles**.

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{[(x+h)^2 + 4(x+h) - 5] - [x^2 + 4x - 5]}{h}$$

$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + 4x + 4h - 5 - x^2 - 4x + 5}{h}$$

$\boxed{2}$

$$= \lim_{h\to 0} \frac{2xh + h^2 + 4h}{h}$$

$$= \lim_{h\to 0} 2x + h + 4 \quad \therefore \frac{dy}{dx} = 2x + 4$$

   b) Find the value of $f'(1)$.

$\boxed{1}$ $x = 1 \ldots \quad \frac{dy}{dx} = 2(1) + 4 = 6$

   c) Find the equation of the tangent line to $f(x)$ at the point $(1,0)$

$\boxed{1}$

$$\frac{6}{1} = \frac{y - 0}{x - 1} \quad \text{OR} \quad 6(x-1) = 1(y-0)$$

$$6x - 6 = y$$

$$\text{OR} \quad 6x - y = 6$$

2. Find the derivative of the following functions using the power rule, sum and/or difference rule, and/or the constant rule.

   a) $y = \dfrac{3}{\sqrt[4]{x}}$

$$y = 3x^{-1/4}$$

$\boxed{4}$

$$\frac{dy}{dx} = -\frac{3}{4}x^{-5/4}$$

$$= \frac{-3}{4x^{5/4}}$$

$$= -\frac{3}{4\sqrt[4]{x^5}}$$

   b) $g(x) = \dfrac{3x^3 - 6x^2 + 2}{x} = \dfrac{3x^3}{x} - \dfrac{6x^2}{x} + \dfrac{2}{x}$

$$g(x) = 3x^2 - 6x + 2x^{-1}$$

$$\frac{dy}{dx} = 6x - 6 - 2x^{-2}$$

$$\frac{dy}{dx} = 6x - 6 - \frac{2}{x^2}$$

$$\frac{dy}{dx} = \frac{6x^3 - 6x^2 - 2}{x^2}$$

velocity $= \frac{\Delta h}{\Delta t}$

3. The equation for the height of an object thrown upward with a velocity of 36 m/s is given by the function $h(t) = 36t - 6t^2$.

a) Find the equation for the instantaneous velocity of the object.

$$\frac{dh}{dt} = 36 - 12t$$

$\boxed{1}$

b) Find the maximum height reached by the object. velocity = zero

$0 = 36 - 12t$

$12t = 36$

$\boxed{2}$

$t = 3$

$h(3) = 36(3) - 6(3)^2$

$h(3) = 54$

$\therefore$ max height is 54 m.

4. Given $f(x) = x^2 + 6x - 16$

a) Find the derivative of $f(x)$.

$\frac{dy}{dx} = f'(x)$

$= 2x + 6$

$\boxed{1}$

b) Find the equation of the tangent line at (2,0).

$f'(2) = 2(2) + 6$

$\frac{dy}{dx} = 10$

$\boxed{2}$

$\frac{10}{1} = \frac{y-0}{x-2}$

$10(x-2) = 1(y-0)$

$10x - 20 = y$

OR

$10x - y = 20$

5. At what points does the curve, $y = 2x^3 + 3x^2 - 36x$ have horizontal tangents?

$$\frac{dy}{dx} = 6x^2 + 6x - 36$$

$$0 = 6x^2 + 6x - 36$$

$$0 = 6(x^2 + x - 6)$$

$$0 = 6(x+3)(x-2) \quad \therefore \; x = -3, \; x = 2$$

3

$$f(-3) = 2(-3)^3 + 3(-3)^2 - 36(-3)$$

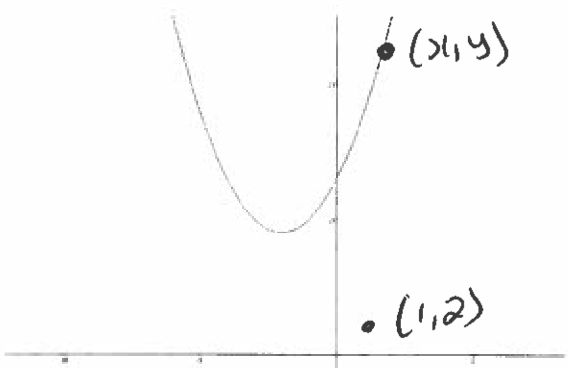
$$f(-3) = 81$$

$$f(2) = -44$$

OR

$(-3, 81)$ and $(2, -44)$

6. Find the slopes of both lines that pass through $(1,2)$ and are tangent to the graph of $y = x^2 + 4x + 13$.



$$m = \frac{y-2}{x-1} \qquad \frac{dy}{dx} = 2x + 4$$

$$m = \frac{(x^2 + 4x + 13) - 2}{x-1}$$

$$m = \frac{x^2 + 4x + 11}{x-1}$$

$$m = \frac{dy}{dx}$$

slope = slope

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$$\frac{x^2 + 4x + 11}{x-1} = \frac{2x+4}{1}$$

$$x^2 + 4x + 11 = (x-1)(2x+4)$$

$$x^2 + 4x + 11 = 2x^2 + 2x - 4$$

$$0 = x^2 - 2x - 15$$

$$0 = (x-5)(x+3)$$

$$x = 5 \qquad x = -3$$

$$\frac{dy}{dx} = 2x + 4 = 2(5) + 4 = 14$$

$$\frac{dy}{dx} = 2(-3) + 4 = -2$$

$P(5, 58)$

$P(-3, 10)$

7. Differentiate the following. Simplify your answers… positive exponents or radicals in simplest form.

a) $y = \sqrt{x}\left(2 - x^2 + 5x^4\right)$, using the product rule. $(fg)' = f'g + fg'$

$f = x^{1/2}$ $\qquad g = 2 - x^2 + 5x^4$

$f' = \frac{1}{2}x^{-1/2}$ $\qquad g' = -2x + 20x^3$

$\boxed{3}$

$$\frac{dy}{dx} = \frac{1}{2}x^{-1/2}(2 - x^2 + 5x^4) + x^{1/2}(-2x + 20x^3)$$

$$= \frac{1}{2}x^{-1/2}\left[(1)(2 - x^2 + 5x^4) + 2x(-2x + 20x^3)\right]$$

$$= \frac{2 - x^2 + 5x^4 - 4x^2 + 40x^4}{2x^{1/2}} = \frac{45x^4 - 5x^2 + 2}{2\sqrt{x}}$$

b) $y = \dfrac{x^2 + 4}{3x - 1}$

$f = x^2 + 4$ $\qquad g = 3x - 1$

$f' = 2x$ $\qquad g' = 3$

$$\frac{dy}{dx} = \frac{f'g - fg'}{g^2}$$

$\boxed{2}$

$$\frac{dy}{dx} = \frac{2x(3x-1) - (x^2+4)(3)}{(3x-1)^2}$$

$$\frac{dy}{dx} = \frac{6x^2 - 2x - 3x^2 - 12}{(3x-1)^2}$$

$$\frac{dy}{dx} = \frac{3x^2 - 2x - 12}{(3x-1)^2}$$

c) $y = \left(4x^3 + 5x\right)^4$

$\boxed{2}$

$$\frac{dy}{dx} = 4(4x^3 + 5x)^3(12x^2 + 5)$$

OR

$$= 4(12x^2 + 5)(4x^3 + 5x)^3$$

$(fg)' = f'g + fg'$

8. Differentiate the following. Simplify your answers… positive exponents.

a) $f(x) = (x-4)^3 (x+2)^2$

$f = (x-4)^3$ $\qquad g = (x+2)^2$

$f' = 3(x-4)^2(1)$ $\qquad g' = 2(x+2)(1)$

$$\frac{dy}{dx} = 3(x-4)^2(x+2)^2 + (x-4)^3(2)(x+2)$$

$$= (x-4)^2(x+2)\left[3(x+2) + (x-4)(2)\right]$$

$\boxed{3}$

$$= (x-4)^2(x+2)\left[3x+6+2x-8\right]$$

$$= (x+2)(x-4)^2(5x-2)$$

b) $f(x) = \dfrac{(3x+1)^2}{(2x+5)}$

$f = (3x+1)^2$ $\qquad g = 2x+5$

$f' = 2(3x+1)(3)$ $\qquad g' = 2$

$f' = 6(3x+1)$

$$\frac{f'g - fg'}{g^2}$$

$$\frac{dy}{dx} = \frac{6(3x+1)(2x+5) - (3x+1)^2(2)}{(2x+5)^2}$$

$\boxed{3}$

$$= \frac{2(3x+1)\left[3(2x+5) - (3x+1)\right]}{(2x+5)^2}$$

$$= \frac{2(3x+1)(6x+15-3x-1)}{(2x+5)^2} = \frac{2(3x+1)(3x+14)}{(2x+5)^2}$$

$\frac{}{10}$

REVIEW:

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1. Find the limits:

a) $\lim_{x\to 9}\frac{\frac{3}{\sqrt{x}}-1}{x-9}\left(\frac{\sqrt{x}}{\sqrt{x}}\right) = \lim_{x\to 9}\frac{3-\sqrt{x}}{\sqrt{x}(x-9)}\left(\frac{3+\sqrt{x}}{3+\sqrt{x}}\right)$

$= \lim_{x\to 9}\frac{9-x}{\sqrt{x}(x-9)(3+\sqrt{x}} = \lim_{x\to 9}\frac{-1}{\sqrt{x}(3+\sqrt{x})}$

$= \frac{-1}{\sqrt{9}(3+\sqrt{9})} = \frac{-1}{18}$

b) $\lim_{x\to 2}\frac{x^2-4}{x^3-2x^2-x+2} = \lim_{x\to 2}\frac{(x+2)(x-2)}{(x^2-1)(x-2)}$

$\begin{array}{r|rrrr} 2 & 1 & -2 & -1 & 2 \\ & & 2 & 0 & -2 \\ \hline & 1 & 0 & -1 & 0 \end{array}$

$= \lim_{x\to 2}\frac{(x+2)}{(x^2-1)} = \frac{2+2}{(2)^2-1} = \frac{4}{3}$

c) $\lim_{n\to\infty}\frac{3n^2+8n+5}{4n^2+1}\ \frac{1/n^2}{1/n^2} = \lim_{n\to\infty}\frac{3+\frac{8}{n}+\frac{5}{n^2}}{4+\frac{1}{n^2}} = \frac{3+0+0}{4+0} = \frac{3}{4}$

2. Given $f(x)=\begin{cases}(x-1)^2+1 & \text{if } x<3 \\ 2 & x=3 \\ 2x-1 & \text{if } x>3\end{cases}$. left of 3 / right of 3

a) Find the limits:

i. $\lim_{x\to 3^-} f(x)$ $= (3-1)^2+1 = 5$

ii. $\lim_{x\to 3^+} f(x)$ $= 2(3)-1 = 5$

iii. $\lim_{x\to 3} f(x) = 5$

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$f(3)=2$ point

b) Is f(x) continuous? Justify by showing your work that demonstrates that conditions of continuity that are met and/or not met.

Not continuous $\lim_{x\to 3} f(x) \neq f(3)$

$5 \neq 2$