

MATH 31

Curves HW #1

Name _____

Date _____

2 each

1. Find the intervals on which each function $f(x)$ is increasing or decreasing given $f'(x)$..

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a) $f'(x) = 5x^2 + 13x + 8$

b) $f'(x) = \frac{x+2}{x^2 - 1}$

$5x^2 + 13x + 8 = 0$

$(5x + 8)(x + 1) = 0$

$x = -\frac{8}{5} \quad x = -1$

$$\frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} = 4$$

	$5x+8$	$x+1$	$f'(x)$	$f''(x)$
$(-\infty, -\frac{8}{5})$	-	-	+	inc
$(-\frac{8}{5}, -1)$	+	-	-	dec
$(-1, \infty)$	+	+	+	inc

Increasing $(-\infty, -\frac{8}{5}) \cup (-1, \infty)$

b) $x+2=0 \quad x=-2$

$x^2-1=0 \quad x=\pm 1$

Decreasing $(-\frac{8}{5}, -1)$

	$x+2$	$x+1$	$x-1$	$f'(x)$	$f''(x)$
$(-\infty, -2)$	-	-	-	-	dec
$(-2, -1)$	+	-	-	+	inc
$(-1, 1)$	+	+	-	-	dec
$(1, \infty)$	+	+	+	+	inc

decreasing $(-\infty, -2) \cup (-1, 1)$

increasing $(-2, -1) \cup (1, \infty)$

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2. Find the intervals of increase and decrease for the functions:

a) $f(x) = x^4 - 4x^3 - 2x^2 + 12x$

b) $f(x) = 2x\sqrt{9-x}$

3 <--> L

a) $f'(x) = 4x^3 - 12x^2 - 4x + 12$

$0 = 4(x^3 - 3x^2 - x + 3)$

$0 = 4(x-1)(x-3)(x+1)$

CN: 1, 3, -1

1	-3	-1	3
1	-2	-3	0
1	-2	-3	0

$x^2 - 2x - 3 = (x-3)(x+1)$

	$x+1$	$x-1$	$x-3$	$f'(x)$	$f(x)$
$(-\infty, -1)$	-	-	-	-	dec
$(-1, 1)$	+	-	-	+	inc
$(1, 3)$	+	+	-	-	dec
$(3, \infty)$	+	+	+	+	inc

increase $(-1, 1) \cup (3, \infty)$ decrease $(-\infty, -1) \cup (1, 3)$

b) $f'(x) = 2(9-x)^{1/2} + 2x\left(\frac{1}{2}\right)(9-x)^{-1/2}(-1)$

$= 2(9-x)^{1/2} - x(9-x)^{-1/2}$

$= (9-x)^{-1/2} [2(9-x) - x]$

$= \frac{18-2x-x}{(9-x)^{1/2}}$

$= \frac{18-3x}{\sqrt{9-x}}$

$= \frac{3(6-x)}{\sqrt{9-x}}$

$f : 2x \quad g : (9-x)^{1/2}$

$f' : 2 \quad g' : \frac{1}{2}(9-x)^{-1/2} (-1)$

	$\sqrt{9-x}$	$6-x$	$f'(x)$	$f(x)$
$(-\infty, 6)$	+	+	+	inc
$(6, 9)$	+	-	-	dec

increase $(-\infty, 6)$ decrease $(6, 9)$

CN: 6

CN: 9

3. Given $f(x) = 4x^3 - 3x^2 - 18x + 5$, $-2 \leq x \leq 3$.

- Find the critical numbers.
- Find the regions of increase and decrease.
- Find the local and/or absolute maximum and minimum values by using the First Derivative Test.
- Sketch $y = f(x)$

$$f'(x) = 12x^2 - 6x - 18$$

$$0 = 12x^2 - 6x - 18$$

$$0 = 6(2x^2 - x - 3)$$

$$0 = 6(2x - 3)(x + 1)$$

$$x = \frac{3}{2}, x = -1$$

	$2x-3$	$x+1$	$f'(x)$	$f(x)$
$(-2, -1)$	-	-	+	inc min
$(-1, \frac{3}{2})$	-	+	-	dec max
$(\frac{3}{2}, 3)$	+	+	+	inc

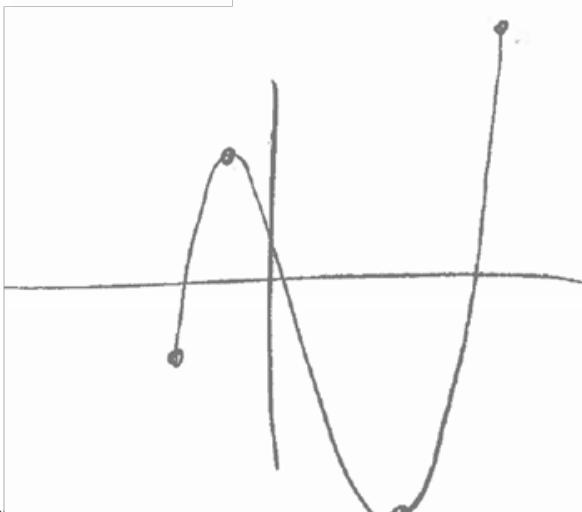
4

$$f(-2) = -3 \quad \text{local min}$$

$$f(-1) = 16 \quad \text{local max}$$

$$f(\frac{3}{2}) = -\frac{61}{4} \quad \text{abs min}$$

$$f(3) = 32 \quad \text{abs max}$$



4. Given $f(x) = \frac{x^2 - x + 1}{x^2 + 1}$,

- Find the critical numbers.
- Find the regions of increase and decrease.
- Find the local and/or absolute maximum and minimum values by using the First Derivative Test.

$$f = x^2 - x + 1 \quad g = x^2 + 1$$

$$f' = 2x - 1 \quad g' = 2x$$

$$y' = \frac{f'g - fg'}{g^2}$$

$$\frac{dy}{dx} = \frac{(2x-1)(x^2+1) - (x^2-x+1)(2x)}{(x^2+1)^2}$$

$$\frac{dy}{dx} = \frac{x^2 - 1}{(x^2 + 1)^2} \quad \frac{dy}{dx} = 0$$

$\frac{dy}{dx} = \text{und}$

$$x^2 - 1 = 0 \quad (x^2 + 1)^2 = 0$$

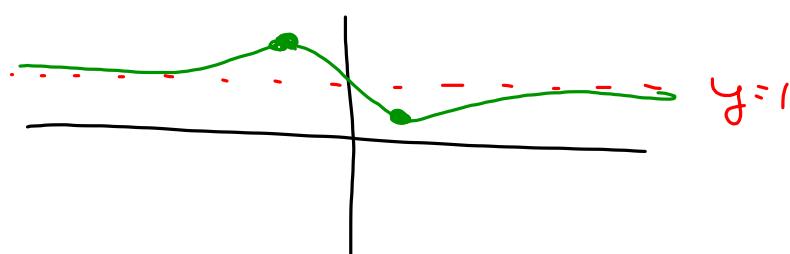
$x = \pm 1$ nothing

$x^2 - 1$	$x+1$	$x-1$	$(x^2+1)^2$	$f'(x)$	$f(x)$
$(-\infty, -1)$	-	-	+	+	INC mtr
$(-1, 1)$	+	-	+	-	DEC
$(1, \infty)$	+	+	+	+	IWC

$$\text{MAX} = f(-1) = \frac{(-1)^2 - (-1) + 1}{(-1)^2 + 1} = \frac{3}{2}$$

$$\text{MIN} = f(1) = \frac{(1)^2 - (1) + 1}{(1)^2 + 1} = \frac{1}{2}$$

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5. Identify the intervals of concave up and concave down for $y = f(x)$ given $f''(x) = \frac{x^2 + x - 12}{x - 1}$.

$$\begin{aligned}f''(x) &= 0 = x^2 + x - 12 \\0 &= (x + 4)(x - 3) \\x &= -4 \quad x = 3\end{aligned}$$

$$f''(x) = \text{undefined}$$

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$$\begin{aligned}0 &= x - 1 \\x &= 1\end{aligned}$$

	$x + 4$	$x - 1$	$x - 3$	f''	$f(x)$
$(-\infty, -4)$	-	-	-	-	CD
$(-4, 1)$	+	-	-	+	CU
$(1, 3)$	+	-	-	-	CD
$(3, \infty)$	+	+	+	+	CU

6. For the curve $y = 2x^3 - 9x^2 + 12x - 2$, find the local maximum and/or minimum values. Justify using the first or second derivative test.

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$$\begin{aligned}
 f'(x) &= 6x^2 - 18x + 12 \\
 0 &= 6(x^2 - 3x + 2) \\
 0 &= (x-2)(x-1) \\
 x=2 &\quad x=1 \\
 \text{"local"} & \\
 \text{minimum } f(2) &= 2(2)^3 - 9(2)^2 + 12(2) - 2 = 2 \text{ ... } (2, 2) \\
 \text{maximum } f(1) &= 2(1)^3 - 9(1)^2 + 12(1) - 2 = 3 \text{ ... } (1, 3) \\
 \text{OR} & \\
 \begin{array}{c|c|c|c|c}
 & x+1 & x-2 & f'(x) & f(x) \\
 \hline
 (-\infty, 1) & - & - & + & \text{inc} \\
 (1, 2) & + & - & - & \text{dec} \\
 (2, \infty) & + & + & + & \text{inc}
 \end{array} &
 \begin{array}{l}
 \rightarrow \max f(1) \\
 \rightarrow \min f(2)
 \end{array}
 \end{aligned}$$

7. For the curve $y = x^3 - 3x^2 - 9x - 5$

- a) Find the intervals of concavity.
 b) Find any inflection points.

$$y'' = 6x - 6$$

$$0 = 6x - 6$$

$$x=1$$

$$\begin{array}{c|c|c|c|c}
 & 6x-6 & f''(x) & f(x) \\
 \hline
 (-\infty, 1) & - & - & \text{CD} \\
 \hline
 (1, \infty) & + & + & \text{CU}
 \end{array}$$

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Concave Down $(-\infty, 1)$

Concave Up $(1, \infty)$

P.I. when $x=1$

$$f(1) = (1)^3 - 3(1)^2 - 9(1) - 5$$

$$f(1) = -16$$