

## CURVE SKETCHING HW #2

1. Given,  $f(x) = \frac{x^2 - 4x - 12}{x^2 - 9}$

Find the vertical and horizontal asymptotes of the curve.

$$y = \lim_{x \rightarrow \infty} \frac{1 - \frac{4}{x} - \frac{12}{x^2}}{1 - \frac{9}{x^2}} = \frac{1 - 0 - 0}{1 - 0} = 1 \quad \text{or} \quad \boxed{y = 1}$$

[3]  $x^2 - 9 = 0$

$\boxed{x=3}$  and  $\boxed{x=-3}$

2. Identify the intervals of increase and decrease for  $y = f(x)$  given  $f'(x) = \frac{x-1}{(x+2)^2}$ .

[3]

	$(x-1)$	$(x+2)^2$	$f'(x)$	$f(x)$
$\rightarrow$	-	+	-	dec
-2 1	-	+	-	dec
$\leftarrow$	+	+	+	inc

decreasing  $(-\infty, -2) \cup (-2, 1)$

increasing  $(1, \infty)$

3. Identify the intervals of concave up and concave down for  $y = f(x)$  given  $f''(x) = \frac{x-10}{\sqrt{x+3}}$ .

[3]

	$\sqrt{x+3}$	$x-10$	$f''(x)$	$f(x)$
$\begin{array}{c} ++ \\ -3 \quad 10 \end{array}$	+	-	-	CD
$(10, \infty)$	+	+	+	CU

4. For the curve  $y = 2x^3 - 9x^2 + 12x - 10$ , find the local maximum and/or minimum values. Justify.

[4]

$$\frac{dy}{dx} = 6x^2 - 18x + 12 = 0$$

$$6(x^2 - 3x + 2) = 0$$

$$(x-2)(x-1) = 0$$

	$(x-1)$	$(x-2)$	$f'(x)$	$f(x)$
$(-\infty, 1)$	-	-	+	inc
$(1, 2)$	+	-	-	dec
$(2, \infty)$	+	+	+	inc

$\nearrow \nwarrow \max f(x)$   
 $\searrow \swarrow \min f(x)$

$$\max f(1) = 2(1)^3 - 9(1)^2 + 12(1) - 10 = -5$$

$$\min f(2) = 2(2)^3 - 9(2)^2 + 12(2) - 10 = -6$$

5. For the curve  $y = 2x^3 + 12x^2 + 18x + 5$

a) Find the intervals of increase and decrease.

$$\frac{dy}{dx} = 6x^2 + 24x + 18 = 6(x^2 + 4x + 3) = 0$$

$$(x+3)(x+1) = 0 \quad \begin{array}{c} + \\ -3 \quad -1 \end{array}$$

	$(x+3)$	$(x+1)$	$f'(x)$	$f(x)$
$(-\infty, -3)$	-	-	+	inc
$(-3, -1)$	+	-	-	dec
$(1, \infty)$	+	+	+	inc

[5]

b) Find the intervals of concavity.

$$y'' = 12x + 24 = 12(x+2) = 0 \quad \begin{array}{c} - \\ -2 \end{array}$$

$$\begin{array}{c} \text{PI } f(-2) \end{array}$$

	$x+2$	$f''(x)$	$f(x)$
$(-\infty, -2)$	-	-	CD
$(-2, \infty)$	+	+	CU

c) Find any inflection points.

$$f(-2) = 2(-2)^3 + 12(-2)^2 + 18(-2) + 5 = 1$$

$$\text{PI } (-2, 1)$$

6. Find the following limits:

a)  $\lim_{x \rightarrow \infty} x^3 + 3x^2 - 4$

$$= \lim_{x \rightarrow \infty} (x^3)(1 + \frac{3}{x} - \frac{4}{x^2})$$

[4]  $= (-\infty)^3(1 + 0 - 0)$   
 $= -\infty$

b)  $\lim_{x \rightarrow -\infty} x^3 + 3x^2 - 4$

$$= \lim_{x \rightarrow -\infty} (x^3)(1 + \frac{3}{x} - \frac{4}{x^2})$$

$$= (-\infty)^3(1 + 0 - 0)$$
  
 $= -\infty$

c)  $\lim_{x \rightarrow \infty} (x-3)(x+2)^2(2-x)$

$$= (\infty)(\infty)^2(-\infty)$$

$$= -\infty$$

d)  $\lim_{x \rightarrow -\infty} (x-3)(x+2)^2(2-x)$

$$= (-\infty)(-\infty)^2(\infty)$$

$$= -\infty$$

7. Find the equations for the vertical and horizontal asymptotes:

a)  $y = \frac{4x-3}{2x+4}$

$$y = \lim_{x \rightarrow \infty} \frac{4 - \frac{3}{x}}{2 + \frac{4}{x}} = \frac{4-0}{2+0} = \frac{4}{2} = 2 \quad \boxed{\text{OR} \quad y=2}$$

$$2x+4=0 \quad \boxed{x=-2}$$

[4]

b)  $y = \frac{x}{x^2+x-6}$

$$y = \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{1 + \frac{1}{x} - \frac{6}{x^2}} = \frac{0}{1+0-0} = \frac{0}{1} = 0$$

$$\boxed{\text{OR} \quad y=0}$$

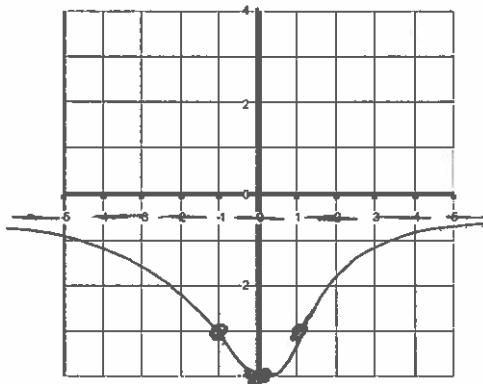
$$x^2+x-6=0$$

$$(x+3)(x-2)=0$$

$$\text{OR} \quad x=-3 \quad x=2$$

8. Given the following properties, sketch the curve for  $f(x)$ .

- [2] The domain is  $\{x \in R\}$ . There are no x-intercepts and  $f(0) = -4$ . There are no vertical asymptotes. The horizontal asymptote is the line  $y = -\frac{1}{2}$ . The interval of decrease is  $(-\infty, 0)$  and the interval of increase is  $(0, \infty)$ .  $f(x)$  is concave downward on  $(-\infty, -1) \cup (1, \infty)$  and is concave upward on  $(-1, 1)$ . The points of inflection are at  $(\pm 1, -3)$ .



7. Sketch the function given:

- [2] The domain is  $\{x \neq \pm 2\}$ . The intercepts are both at 0 and  $f(0)$  is a local minimum. The vertical asymptotes are  $x = 2$  and  $x = -2$ . The horizontal asymptote is the line  $y = -4$ . The interval of increase is  $(0, 2) \cup (2, \infty)$ . The interval of decrease is  $(-\infty, -2) \cup (-2, 0)$ .  $f(x)$  is concave upward on  $(-2, 2)$ .  $f(x)$  is concave downward on  $(-\infty, -2) \cup (2, \infty)$ . There are no points of inflection.

