Math 31 Name ________________ $\frac{}{20}$

Exponential and Logarithmic: HW #2 Date ________________

[4] 1. Find the derivatives of

a) $y = 5^{\frac{-1}{x}}$

$$\frac{dy}{dx} = 5^{-1/x}(\ln 5)\left(\frac{1}{x^2}\right)$$

$$= \frac{\ln 5}{\sqrt[x]{5}\, x^2}$$

b) $y = 3^{x^4}$

$$\frac{dy}{dx} = 3^{x^4} \ln 3\,(4x^3)$$

[6] 2. Use logarithmic differentiation to find the derivative of

a) $y = x^{x^4}$

$$\ln y = x^4 \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = 4x^3 \ln x + x^4\left(\frac{1}{x}\right)$$

$$\frac{dy}{dx} = y\left[4x^3 \ln x + x^3\right]$$

$$\frac{dy}{dx} = x^{x^4}\left[x^3(4\ln x + 1)\right]$$

$$= x^{x^4+3}(4\ln x + 1)$$

b) $y = x^2 e^x \sqrt{x^2 + x - 8}$

$$\ln y = \ln x^2 + \ln e^x + \ln(x^2+x-8)^{1/2}$$

$$\ln y = 2\ln x + x + \frac{1}{2}\ln(x^2+x-8)$$

$$\frac{1}{y}\frac{dy}{dx} = 2\left(\frac{1}{x}\right) + 1 + \frac{1}{2}\left[\frac{2x+1}{x^2+x-8}\right]$$

$$\frac{dy}{dx} = y\left[\qquad\right]$$

$$\frac{dy}{dx} = x^2 e^x \sqrt{x^2+x-8}\left[\frac{2}{x} + 1 + \frac{2x+1}{2(x^2+x-8)}\right]$$

[10] 3. Discuss the curve $y = \ln(16 - x^2)$ under the following headings.

a) Domain. 1

ln (pos) ∴ $16 - x^2 = \text{pos}$

domain: $(-4, 4)$

b) Intercepts. 2

$x = 0$
$y = \ln 16$

$y = 0$
$0 = \ln(16 - x^2)$, $e^0 = 16 - x^2$
$x^2 = 15$
$x = \pm\sqrt{15}$

c) Symmetry. 1

$f(-x) = \ln(16 - (-x)^2)$
$= \ln(16 - x^2)$
$= f(x)$
∴ even function

d) Asymptotes. 1

$x = 4$ $x = -4$

$$y = \ln(16 - x^2)$$

e) Intervals of increase or decrease.

$$\frac{dy}{dx} = \left(\frac{1}{16-x^2}\right)(-2x) = \frac{-2x}{16-x^2}$$

	$-2x$	$16-x^2$	$f'(x)$	$f(x)$
$(-4, 0)$	$+$	$+$	$+$	inc
$(0, 4)$	$-$	/	$-$	dec

$c = 0, \pm 4$

2

increasing $(-4, 0)$

decreasing $(0, 4)$

f) Regions of concavity.

1 $$y'' = \frac{(-2)(16-x^2) - (-2x)(-2x)}{(16-x^2)^2} = \frac{-32 + 2x^2 - 4x^2}{(16-x^2)^2}$$

$$= \frac{-2x^2 - 32}{(16-x^2)^2} = \frac{-2(x^2+16)}{(16-x^2)^2}$$

$\therefore$ $f''(x) < 0$ always

$\therefore$ CD $(-4, 4)$

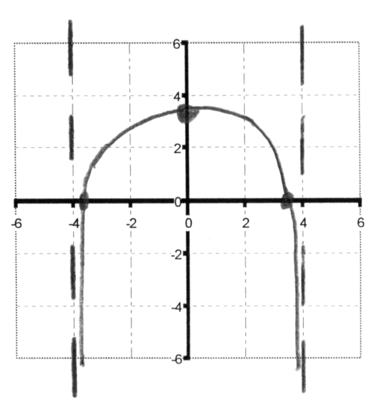
g) Local maximum and minimum values.

$f'(c) = 0$, $c = 0$

1 $f''(0) < 0$, CD $\therefore$ local max $(0, \ln 16)$

h) Sketch.

1



Math 31 Name ________________

Exponential and Logarithmic: HW #2 Date ________________

1. Use logarithmic differentiation to find the derivative of $y = x^2 e^x \sqrt{x^2 + x - 8}$

13

$\ln y = \ln(x^2) + \ln(e^x) + \ln(x^2+x-8)^{1/2}$

$\ln y = 2\ln x + x + \frac{1}{2}\ln(x^2+x-8)$

$\boxed{3}$ $\left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right) = 2\left(\frac{1}{x}\right) + (1) + \frac{1}{2}\left(\frac{1}{x^2+x-8}\right)(2x+1)$

$\frac{dy}{dx} = x^2 e^x \sqrt{x^2+x-8}\left(\frac{2}{x} + 1 + \frac{2x+1}{2(x^2+x-8)}\right)$

2. Discuss the curve $y = \ln(16 - x^2)$ under the following headings.

a) Domain.

$16 - x^2 > 0$

$16 > x^2$

1 $(-4, 4)$

b) Intercepts.

2

$x=0$, $y = \ln 16$ ... $(0, \ln 16)$

$y=0$, $0 = \ln(16-x^2)$

$e^0 = 16 - x^2$

$1 = 16 - x^2$

$x^2 = 15$ $\therefore$ $(\pm\sqrt{15}, 0)$

c) Symmetry.

$f(-x) = \ln(16-(-x)^2)$

1 $f(-x) = \ln(16-x^2)$

$f(-x) = f(x)$

$\therefore$ sym y-axis

"even function"

d) Asymptotes.

$\lim_{x\to 4^-} \ln(16-x^2) = -\infty$

1

$\lim_{x\to -4^+} \ln(16-x^2) = -\infty$

$\therefore$ $x=4$, $x=-4$