Math 31 Name: Key

Quiz: Chain Rule & Implicit Differentiation

1. Differentiate the following. Be sure to simplify.

a) $y = (7x^2 + 3)^5$

$$\frac{dy}{dx} = 5(7x^2+3)^4 \cdot \frac{d}{dx}(7x^2+3)$$

$$= 5(7x^2+3)^4 (14x)$$

$$= 70x(7x^2+3)^4$$

[5]

b) $f(x) = (2x + 1)^3(5x^2 + 4)$

$f = (2x+1)^3 \qquad g = (5x^2+4)$

$f' = 3(2x+1)^2(2) \qquad g' = 10x$

$f' = 6(2x+1)^2$

$(fg)' = f'g + fg'$

$$f'(x) = 6(2x+1)^2(5x^2+4) + (2x+1)^3(10x)$$

$$f'(x) = 2(2x+1)^2\left[3(5x^2+4) + 5x(2x+1)\right]$$

$\therefore 15x^2 + 20 + 10x^2 + 5x$

$$f'(x) = 2(2x+1)^2(25x^2+5x+12)$$

1. continued... Differentiate the following. Be sure to simplify.

c) $y = \frac{(3x^2-5)^5}{\sqrt{6x+1}}$ [4]

$\frac{f'g - fg'}{g^2}$

$f = (3x^2-5)^5$

$f' = 5(3x^2-5)^4(6x)$

$f' = 30x(3x^2-5)^4$

$g = (6x+1)^{1/2}$

$g' = \frac{1}{2}(6x+1)^{-1/2}(6)$

$g' = 3(6x+1)^{-1/2}$

$$\frac{dy}{dx} = \frac{30x(3x^2-5)^4(6x+1)^{1/2} - (3x^2-5)^5(3)(6x+1)^{-1/2}}{\left[(6x+1)^{1/2}\right]^2}$$

$$\frac{dy}{dx} = \frac{3(3x^2-5)^4(6x+1)^{-1/2}\left[10x(6x+1) - (3x^2-5)\right]}{(6x+1)}$$

$$\frac{dy}{dx} = \frac{3(3x^2-5)^4\left[60x^2+10x-3x^2+5\right]}{(6x+1)^{3/2}}$$

$$\frac{dy}{dx} = \frac{3(3x^2-5)^4(57x^2+10x+5)}{\sqrt{(6x+1)^3}}$$

2. Given: $\left[2x^3 + 6x^2 - 8y^3 + 5y^2 = 5\right] \frac{d}{dx}$

a) Solve for $\frac{dy}{dx}$. Be sure to simplify.

$$6x^2 + 12x - 24y^2\frac{dy}{dx} + 10y\frac{dy}{dx} = 0$$

$$6x^2 + 12x = \left(24y^2 - 10y\right)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{6x(x+2)}{2y(12y-5)}$$

[5]

$$\frac{dy}{dx} = \frac{3x(x+2)}{y(12y-5)}$$

b) Find the slope at P(1,1).

$$m = \frac{3(1)(1+2)}{(1)(12(1)-5)}$$

$$m = \frac{3(3)}{7}$$

$$m = \frac{9}{7}$$

3. Differentiate the following. Be sure to simplify.

$\left[3x^2y - 4xy^2 = 20\right] \frac{d}{dx}$

[4]

$$3\left[2xy + x^2(1)\frac{dy}{dx}\right] - 4\left[(1)y^2 + x(2y)\frac{dy}{dx}\right] = 0$$

$$6xy + 3x^2\frac{dy}{dx} - 4y^2 - 8xy\frac{dy}{dx} = 0$$

$$6xy - 4y^2 = 8xy\frac{dy}{dx} - 3x^2\frac{dy}{dx}$$

$$6xy - 4y^2 = \left[8xy - 3x^2\right]\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2y(3x-2y)}{x(8y-3x)}$$

$$\frac{4y^2-6xy}{3x^2-8xy} = \frac{6xy-4y^2}{8xy-3x^2}$$

$$\frac{2y(2y-3x)}{x(3x-8y)} = \frac{2y(3x-2y)}{x(8y-3x)}$$

4. Find y'' in simplest form given $x^4 + y^4 = 10$

[4]

$$\frac{d}{dx}(x^4 + y^4 = 10)$$

$$4x^3 + 4y^3\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{4x^3}{4y^3} \quad \therefore \frac{dy}{dx} = \frac{-x^3}{y^3}$$

$f = -x^3$ $\qquad$ $g = y^3$

$f' = -3x^2$ $\qquad$ $g' = 3y^2 \cdot \frac{dy}{dx}$

$$= 3y^2\left(\frac{-x^3}{y^3}\right) = -\frac{3x^3}{y}$$

$$y'' = \frac{(-3x^2)(y^3) - (-x^3)\left(\frac{-3x^3}{y}\right)}{[y^3]^2}\left(\frac{y}{y}\right)$$

$$y'' = \frac{-3x^2y^4 - 3x^6}{y^7}$$

$$y'' = \frac{-3x^2(y^4 + x^4)}{y^7}$$

... $x^4 + y^4 = 10$

$$y'' = \frac{-3x^2(10)}{y^7}$$

$$y'' = \frac{-30x^2}{y^7}$$