Math 31 Name Key.

Differential Equations & Area Exam

1. Find the indefinite integral.

a) $\int 2x^3\,dx$

$= 2\left(\frac{1}{4}x^4\right) + c$

$= \frac{1}{2}x^4 + c$

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b) $\int \left(x^2 + 3e^{-x} + 4\sin x\right)dx$.

$= \frac{1}{3}x^3 + 3e^{-x}\left(\frac{1}{-1}\right) + 4(-\cos x) + c$

$= \frac{1}{3}x^3 - 3e^{-x} - 4\cos x + c$

$= \frac{1}{3}x^3 - \frac{3}{e^x} - 4\cos x + c$

2. Determine the integral for the existing conditions.

a) $\int (2x+1)dx$ and $y = 8$ when $x = -2$.

$y = 2\left(\frac{1}{2}x^2\right) + x + c$

$y = x^2 + x + c$

$8 = (-2)^2 + (-2) + c$

$8 = 4 - 2 + c$

$c = 6$

$y = x^2 + x + 6$

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OR

$\int (2x+1)dx = \frac{(2x+1)^2}{(2)(2)} + c$

$8 = \frac{[2(-2)+1]^2}{4} + c$

$8 = \frac{9}{4} + c \quad \therefore c = \frac{23}{4}$

$F(x) = \frac{1}{4}(2x+1)^2 + \frac{23}{4}$

b) Suppose $F'(x) = 3\sqrt{x} - 6$ and the point (4,2) is on $y = F(x)$. Find $F(x)$.

$F(x) = \int 3(x)^{1/2} - 6$

$= 3\left(\frac{2}{3}x^{3/2}\right) - 6x + c$

$F(x) = 2x^{3/2} - 6x + c$

$2 = 2(4)^{3/2} - 6(4) + c$

$2 = 16 - 24 + c$

$10 = c$

$F(x) = 2x^{3/2} - 6x + 10$

$= 2\left(x^{3/2} - 3x + 5\right)$

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3. Determine the following indefinite integrals.

a) $\int\left(6x^2\left(4x^3+9\right)^9\right)dx$

$u = 4x^3+9$

$\frac{du}{dx} = 12x^2 \quad \therefore dx = \frac{du}{12x^2}$

$\int 6x^2(u)^9 \frac{du}{12x^2}$

$= \int \frac{1}{2}u^9\, du$

$= \frac{1}{2}\left(\frac{1}{10}u^{10}\right)+c$

$= \frac{1}{20}(4x^3+9)^{10}+c$

$\int 6x^2(4x^3+9)^9\, dx$

$= 6x^2\left(\frac{4x^3+9}{10}\right)^{10}\left(\frac{1}{12x^2}\right)$

$= \frac{1}{20}(4x^3+9)^{10}+c$

b) $\int\left(e^{\cos x}\sin x\right)dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x \quad \therefore dx = \frac{du}{-\sin x}$

$= \int e^u \sin x \frac{du}{-\sin x}$

$= \int -e^u\, du$

$= -e^u + c \rightarrow = -e^{\cos x}+c$

$\int e^{\cos x}\sin x\, dx = e^{\cos x}\sin x\left(\frac{1}{-\sin x}\right)+c$

$= -e^{\cos x}+c$

4. A pebble is tossed upward at 25 m/s from the edge of a bridge 46 m above the river below. How many seconds elapse between the toss and the splash? Round your answer off to the nearest tenth of a second.

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$a = -9.8\ m/s^2$

$V = \int -9.8\, dt$

$V = -9.8t + c \quad$ ooo $\quad V = 25,\ t=0$

$\therefore V = -9.8t + 25$

$s = \int -9.8t + 25\, dt$

$s = -4.9t^2 + 25t + C \quad$ ooo $\quad s = 46,\ t=0$

$\therefore s = -4.9t^2 + 25t + 46$

$t = ?$ when $s = 0$

$0 = -4.9t^2 + 25t + 46$

$t \neq -1.4,\ \boxed{t = 6.5s}$

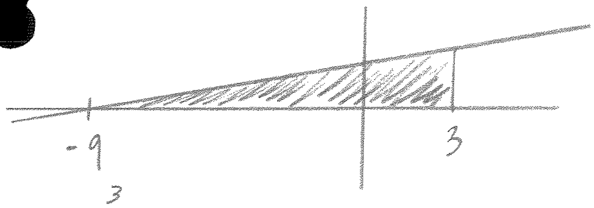
$t = \frac{-25 \pm \sqrt{25^2 - 4(-4.9)(46)}}{2(-4.9)}$

$t = \frac{-25 \pm \sqrt{1526.6}}{-9.8}$

5. Sketch and find the area of the region

[redacted]

a) between the x-axis and the curve $y = \frac{2}{3}x + 6$, from $x = -9$ to $x = 3$.



$$A = \int_{-9}^{3} \frac{2}{3}x + 6 \, dx$$

$$= \frac{1}{3}x^2 + 6x \Big]_{-9}^{3}$$

$$= \left[\frac{1}{3}(3)^2 + 6(3)\right] - \left[\frac{1}{3}(-9)^2 + 6(-9)\right]$$

$$= (3 + 18) - (27 - 54)$$

$$= 21 + 27$$

$$= 48 \text{ units}^2$$

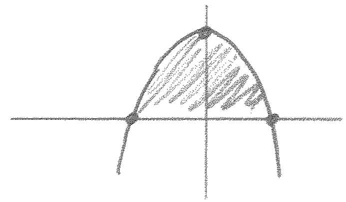
b) bounded by the curve $y = 8 - \frac{1}{2}x^2$ and the x-axis.

$$0 = 8 - \frac{1}{2}x^2$$

$$\frac{1}{2}x^2 = 8$$

$$x^2 = 16$$

$$x = \pm 4$$



$$A = \int_{-4}^{4} 8 - \frac{1}{2}x^2 \, dx$$

$$= \left[8x - \frac{1}{6}x^3\right]_{-4}^{4}$$

$$= \left[8(4) - \frac{1}{6}(4)^3\right] - \left[8(-4) - \frac{1}{6}(-4)^3\right]$$

$$= \left[32 - \frac{32}{3}\right] - \left[-32 + \frac{32}{3}\right]$$

$$= \frac{96}{3} - \frac{32}{3} + \frac{96}{3} - \frac{32}{3}$$

$$= \frac{128}{3} \text{ units}^2$$

[redacted] Evaluate exactly: $\int_1^3 \frac{1 + 3x}{x^2} dx$

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$$= \int_1^3 \frac{1}{x^2} + \frac{3x}{x^2} \, dx$$

$$= \int_1^3 x^{-2} + \frac{3}{x} \, dx$$

$$= -x^{-1} + 3\ln x \Big]_1^3$$

$$= (-3^{-1} + 3\ln 3) - (-1^{-1} + 3\ln 1)$$

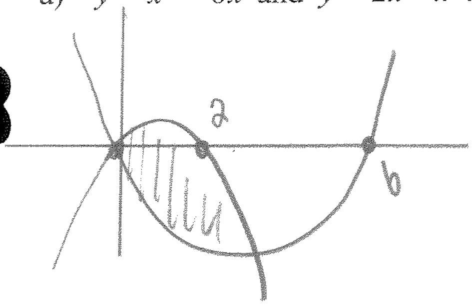
$$= -\frac{1}{3} + \ln 27 + 1 + \ln 1 \qquad \ln e^0 = 0$$

[redacted] $\frac{2}{3} + \ln 27$

$\boxed{6}$

7. Find the area between the curves, exact values. Include a sketch.

a) $y = x^2 - 6x$ and $y = 2x - x^2$.



$$x^2 - 6x = 2x - x^2$$

$$2x^2 - 8x = 0$$

$$2x(x-4) = 0$$

$$x = 0 \quad x = 4$$

$$A = \int_0^4 (2x - x^2) - (x^2 - 6x)\, dx$$

$$= \int_0^4 8x - 2x^2\, dx$$

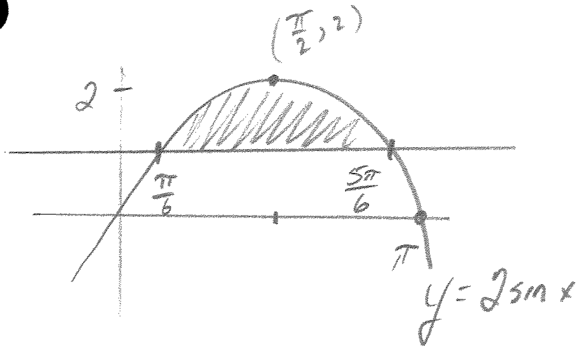
$$= 4x^2 - \frac{2}{3}x^3 \Big]_0^4$$

$$= \left[4(4)^2 - \frac{2}{3}(4)^3\right] - [0]$$

$$= 64 - \frac{128}{3}$$

$$= \frac{64}{3} \text{ units}^2$$

b) $y = 2\sin x$ and $y = 1$ from $x = \dfrac{\pi}{6}$ to $x = \dfrac{5\pi}{6}$, *exact values.*



$$2\sin x = 1$$

$$\sin x = \frac{1}{2} \quad \text{on} \quad x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$A = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 2\sin x - 1\, dx = -2\cos x - x \Big]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$= \left[-2\cos\frac{5\pi}{6} - \frac{5\pi}{6}\right] - \left[-2\cos\frac{\pi}{6} - \frac{\pi}{6}\right]$$

$$= -2\left(-\frac{\sqrt{3}}{2}\right) - \frac{5\pi}{6} + 2\left(\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}$$

$$= \sqrt{3} - \frac{5\pi}{6} + \sqrt{3} + \frac{\pi}{6} = 2\sqrt{3} - \frac{2\pi}{3} \text{ OR } \frac{6\sqrt{3} - 2\pi}{3} \text{ units}^2$$