Math 31 Name: ____________________

Limit Unit Assignment

1. What is the difference between a secant line and a tangent line?

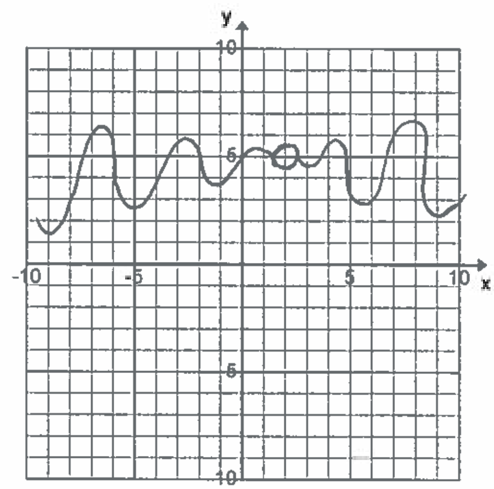
secant cross function at two points
tangent touches function at one point

2. What is the difference between how we find the slope of a secant line to a curve and how we find the slope of a tangent line to a curve?

secant $= \frac{\Delta y}{\Delta x}$ tangent $= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$

$= \frac{y_2 - y_1}{x_2 - x_1}$

3. Draw a function that approaches 5 from the left side of 2 and approaches 5 from the right side of 2 but the point (2,5) does not exist on the function.



4. What is a limit?

Limit is the function value approached from both the left and right side of a given x-value... not at x but the value of the function that we get as we approach x...

5. The points P (1,5) and Q (-1,-3) lie on the parabola defined by $y = x^2 + 4x$.

a) Find the slope of the tangent line to the parabola at point P using the format:

$$\lim_{x \to a} \frac{f(a) - f(x)}{a - x}$$

$a \to x$

b) Find the slope of the tangent line to the parabola at Q using the format: $m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

c) Find the equation of the tangent line at P or at Q in general form: *Ax+By+C=0*

a) @ $P(1,5)$

$P(a, f(a))$

$$m = \lim_{a \to 1} \frac{f(a) - f(1)}{a - 1}$$

$$m = \lim_{a \to 1} \frac{a^2 + 4a - 5}{a - 1}$$

$$m = \lim_{a \to 1} \frac{(a+5)(a-1)}{a-1}$$

$$m = 1 + 5$$

$$m = 6$$

b) $Q(-1,-3)$

$P(-1+h, f(-1+h))$

OR

$P(h-1, f(h-1))$

$$m = \lim_{h \to 0} \frac{f(h-1) - f(1)}{h}$$

$$m = \lim_{h \to 0} \frac{[(h-1)^2 + 4(h-1)] - [-3]}{h}$$

$$m = \lim_{h \to 0} \frac{h^2 - 2h + 1 + 4h - 4 + 3}{h}$$

$$m = \lim_{h \to 0} \frac{h^2 + 2h}{h} \quad \ldots \quad \frac{h^2}{h} + \frac{2h}{h} \quad \text{OR} \quad \frac{h(h+2)}{h}$$

$$m = \lim_{h \to 0} h + 2$$

$$m = 2$$

c) $Q(-1,-3)$

$m = 2$

$$\frac{2}{1} = \frac{y+3}{x+1}$$

$$2x + 2 = y + 3$$

$$2x - y - 1 = 0$$

$P(1,5)$

$m = 6$

$$\frac{6}{1} = \frac{y-5}{x-1}$$

$$6x - 6 = y - 5$$

$$6x - y - 1 = 0$$

6. Evaluate the following limits, if they exist.

a) $\lim_{x \to 2} x^2 - x - 6$ not ÷ zero

$= (2)^2 - (2) - 6$

$= -4$

b) $\lim_{x \to -5} \frac{x^2 - 25}{x^2 + 10x + 25}$ $= \lim_{x \to -5} \frac{(x+5)(x-5)}{(x+5)(x+5)}$

$= \lim_{x \to -5} \frac{x-5}{x+5}$

$= \frac{-10}{0}$ ∴ undefined

c) $\lim_{x \to 0} \frac{\frac{6}{x+3} - 2}{x} \frac{(x+3)}{(x+3)}$ $= \lim_{x \to 0} \frac{6 - 2x - 6}{x(x+3)}$

$= \lim_{x \to 0} \frac{-2x}{x(x+3)}$

$= \lim_{x \to 0} \frac{-2}{x+3}$

$= -\frac{2}{3}$

d) $\lim_{x\to 2}\frac{x^2-4}{x^3-2x^2-x+2}$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -1 & 2 \\ & & 2 & 0 & -2 \\ \hline & 1 & 0 & -1 & 0 \end{array}$$

$= \lim_{x\to 2} \frac{(x+2)(x-2)}{(x-2)(x^2-1)}$

$= \lim_{x\to 2} \frac{x+2}{x^2-1} \quad \frac{x+2}{(x-1)(x+1)}$

$= \frac{4}{3}$

e) $\lim_{x\to -3}\frac{x^3+27}{x^2+4x+3} = \lim_{x\to -3} \frac{(x+3)(x^2-3x+9)}{(x+3)(x+1)}$

$= \frac{(-3)^2-3(-3)+9}{-3+1}$

$= -\frac{27}{2}$

f) $\lim_{n\to\infty}\frac{3n^2+8n+5}{4n^2+1} \quad \frac{1/n^2}{1/n^2} = \lim_{n\to\infty} \frac{3+\frac{8}{n}+\frac{5}{n^2}}{4+\frac{1}{n^2}}$

$= \frac{3+0+0}{4+0}$

$= \frac{3}{4}$

$f(5)=\sqrt{5-4}$ $f(5+h)=\sqrt{5+h}$

7. Find the slope on the function: $f(x)=\sqrt{x-4}$ when $x=5$. $f(5)=\sqrt{1}$ $f(5+h)=\sqrt{h+1}$

$f(5)=1$

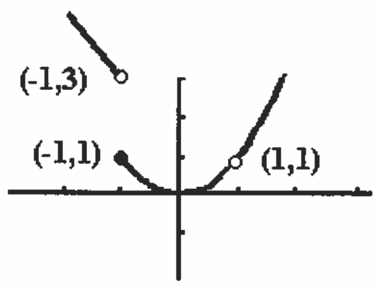
$$m=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$m=\lim_{h\to 0}\left(\frac{\sqrt{h+1}-1}{h}\right)\left(\frac{\sqrt{h+1}+1}{\sqrt{h+1}+1}\right)$$

$$m=\lim_{h\to 0}\frac{h+1-1}{h(\sqrt{h+1}+1)}$$

$$m=\frac{1}{\sqrt{0+1}+1}=\frac{1}{2}$$

8. Given the graph of f(x).



Find the following limits, if they exist. If a limit does not exist, explain why.

a) $\lim_{x\to -1^-} f(x)=3$

LEFT...

b) $\lim_{x\to -1^+} f(x)=1$

RIGHT

c) $\lim_{x\to -1} f(x)=$ undefined

not same value from the left and right sides

d) $\lim_{x\to 1} f(x)=1$

both left and right sides approach same value.

9. Given $f(x)=\begin{cases}(x-1)^2+1 & \text{if } x<3 \\ 2 & x=3 \\ 2x-1 & \text{if } x>3\end{cases}$. ← left of 3 … open circle; ← at 3 … $f(3)=2$; ← right of 3 … ~~closed~~ open circle

a) Find the limits:

i. $\lim_{x\to 3^-} f(x)$

$= (3-1)^2+1$

$= 5$

ii. $\lim_{x\to 3^+} f(x)$

$= 2(3)-1$

$= 5$

iii. $\lim_{x\to 3} f(x)$

$= 5$ same value from left and right side.

b) Is f(x) continuous? Justify by showing your work that demonstrates that conditions of continuity that are met and/or not met.

I. limit exists ✓ $\lim_{x\to 3} f(x) = 5$

II. point exists ✓ $f(3)=2$

III $\lim_{x\to 3} f(x) \neq f(3)$

$5 \neq 2$