Derivative of the Exponential Function With Base e

Objectives: Find the derivative of exponential functions.

Estimations of *e*

If you began walking at 1 km/h and then doubled your speed over a one-minute interval, you would be walking at 2 km/h. But suppose you increased your speed by 50% every half-minute. How fast would you be walking at the end of one minute?

Suppose you increased your speed by 25% every quarter-minute (15 seconds). What would your speed be at the end of one minute? Remember, your speed would be 1.25 times as fast every quarter-minute. Complete the chart below

Time Elapsed (s)	0	15	30	45	60
Speed (km/h)					

Generate an expression to find your speed at the end of one minute.

Suppose you increased your speed by $\frac{1}{10}$ for every tenth of a minute. What would your speed be at the end of one minute?

Complete the table for each increase in speed for and equal portion of a minute.

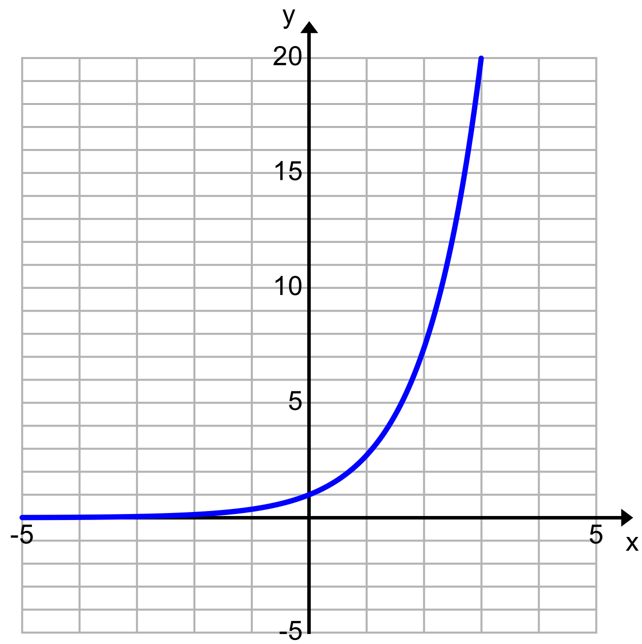
Increase in speed	Speed at the end of 1 minute
$\frac{1}{10}$	
$\frac{1}{1000}$	
$\frac{1}{100000}$	
$\frac{1}{10000000}$	
$\frac{1}{1000000000}$	

Derivatives of $y = e^x$

e can be defined as: $e = \lim_{n \to \infty} \left[1 + \frac{1}{n} \right]^n$

Investigate:

Why is *e* such a special number?



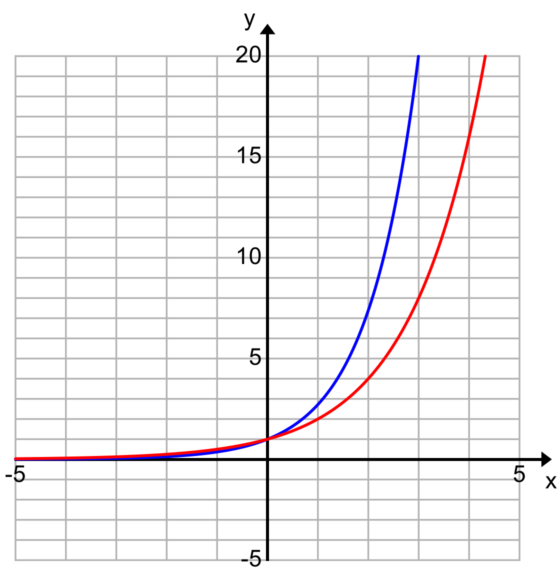
Use your calculator to sketch $y = e^x$.

Find the values of e^x at x = 1,3,5

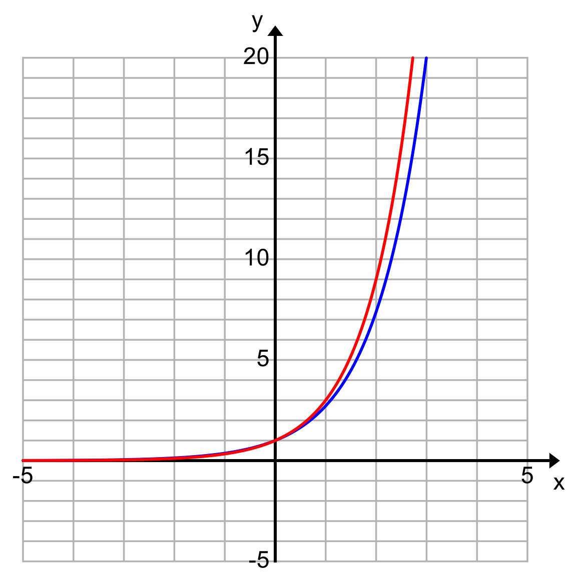
Find the derivative of $y = e^x$ at x = 1,3,5 using your calculator.

State the value of $\frac{dy}{dx} e^x$

$y = 2^x$ and $y = e^x$



$y = 3^x$ and $y = e^x$



Chain Rule: $\boldsymbol{f(x) = e^u}$ then $\boldsymbol{f'(x) = e^u \cdot \frac{du}{dx}}$

1. Differentiate
   a) $y = x^3 e^x$
   b) $y = e^{x^2}$
   c) $y = x^5 e^{x^5}$

2. Find the absolute maximum value of the function $f(x) = xe^{-x}$ using the first or second derivative test.

3. Given: $f(x) = e^{-x^2}$
   a) Sketch the function.
   b) Differentiate the function.

Homework: Page 366 # 1, 4 (a,b,d,g,h,k,l) , 5, 8, 10, 11(a,b)
Calculus, A First Course. McGraw-Hill Ryerson Limited, 1989.

1. Simplify.

(a) $\frac{2}{e^{-x}}$ (b) $(e^x)^4$

(c) $e^{1-x}e^{3x}$ (d) e^xe^{-x}

(e) $e^{2x}(1 - 5e^{3x})$ (f) $\frac{6e^{8x}}{e^{3x}}$

4. Differentiate.

(a) $y = 2e^{-x}$ (b) $y = x^4e^x$

(c) $y = e^{2x}\sin 3x$ (d) $y = e^{\sqrt{x}}$

(e) $y = e^{\tan x}$ (f) $y = \tan(e^x)$

(g) $y = \frac{e^x}{x}$ (h) $y = \frac{e^x}{1 - e^{2x}}$

(i) $y = e^{\sin(x^2)}$ (j) $y = xe^{\cot 4x}$

(k) $y = (1 + 5e^{-10x})^4$ (l) $y = \sqrt{x + e^{1-x^2}}$

5. Find the equation of the tangent line to the curve $y = 1 + xe^{2x}$ at the point where $x = 0$.

8. Find the intervals of increase and decrease for the function $f(x) = x^2e^{-x}$.

9. Find the absolute minimum value of the function $f(x) = \frac{e^x}{x}, x > 0$.

10. For the function $f(x) = xe^x$, find

(a) the absolute minimum value,

(b) the intervals of concavity,

(c) the inflection point.

11. Evaluate.

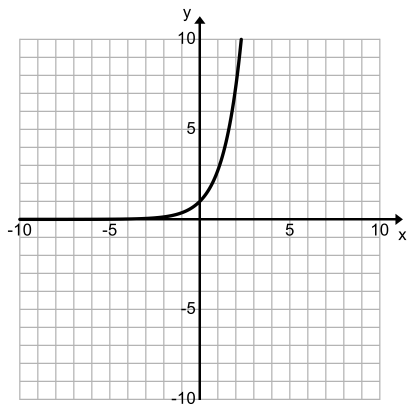
(a) $\lim_{x\to\infty} e^{-x}$ (b) $\lim_{x\to-\infty} e^{-x}$ (c) $\lim_{t\to\frac{\pi}{2}^+} e^{\tan t}$

The Natural Logarithm

Objectives:

Use natural logarithms "$y = log_e x$ *is the same as* $y = lnx$" to simplify, change forms, solve equations.

Warm up: Given the function $y = e^x$ sketch $y = \ln x$



$\ln x = \log_e x$ is called the natural logarithm.

1. Simplify the following:
   a) $\ln e^x$ b) $e^{\ln x}$ c) $\ln e$ d) $\ln 1$

2. Solve for x in the following:
   a) $\ln x = 5$ b) $e^x = 20.086$ c) $e^{3-2x} = 4$

3. Sketch the graphs of the following functions.
   a) $y = -\ln x$ b) $y = \ln(-x)$

4. Express $\frac{2}{3}\ln x - 4\ln y + \ln(x+1)$ as a single logarithm.

5. Find the domain of the function $f(x) = \ln(16 - x^2)$

6. Find $\lim_{x \to 4^-} \ln(16 - x^2)$

Homework: Page 375 #3,4,5,6,9(a,b),10.

The Derivative of Logarithmic Functions

Objectives: Find Derivatives of Logarithmic functions.

Use the exponential form and implicit differentiation to find the derivative of $y = \ln x$

"$y = lnx$ is the same as $y = \log_e x$ is the same as $e^y = x$"

SUMMARY: $y = \ln(u) \ldots \frac{dy}{dx} =$

Examples:

1. Differentiate

   a) $y = x^2 \ln x$

   b) $y = \ln(x^2 + 1)$

   c) $y = (\ln x)^3$

   d) $y = x \ln x$

   e) $y = \ln \frac{x}{\sqrt{x+1}}$

   f) $y = \ln|x|$

2. Find the derivative of $y = \log_3 x$

3. Develop a formula for finding $\frac{d}{dx} \log_b x$ using what you discovered above.

4. Find $f'(x)$ if $f(x) = \log(x^2 + x)$

Homework: Page 383 #1(a,b,c,d,e,g,h,j,k,l) 3, 5(a,b,d)
Calculus, A First Course. McGraw-Hill Ryerson Limited, 1989.

B **1.** Differentiate.

(a) $f(x) = x^2 \ln x$

(b) $f(x) = \sqrt{\ln x}$

(c) $g(x) = \ln(x^3 + 1)$

(d) $g(x) = \ln(5x)$

(e) $y = \sin(\ln x)$

(f) $y = \ln(\sin x)$

(g) $y = \dfrac{\ln x}{x^3}$

(h) $y = (x + \ln x)^3$

(i) $y = \ln|2x + 1|$

(j) $y = \ln\left(\dfrac{x + 1}{x - 1}\right)$

(k) $y = \ln\sqrt{\dfrac{x}{2x + 3}}$

(l) $y = \ln\dfrac{x}{\sqrt{x^2 + 1}}$

(m) $y = \ln(\sec x + \tan x)$

(n) $y = \tan[\ln(1 - 3x)]$

2. (a) If $f(x) = \ln(\ln x)$, find $f'(x)$.

(b) Find the domains of f and f'.

3. Find the derivative of each function.

(a) $f(x) = \log_2(x^2 + 1)$

(b) $g(x) = x \log_{10} x$

(c) $F(x) = \log_5(3x - 8)$

(d) $G(x) = \dfrac{1 + \log_3 x}{x}$

5. Find the equation of the tangent line to each curve at the given point.

(a) $y = \ln(x - 1)$, $(2, 0)$

(b) $y = x^2 \ln x$, $(1, 0)$

(c) $y = 10^x$, $(1, 10)$

(d) $y = \log_{10} x$, $(100, 2)$

The Derivative of Exponential Functions

Objective: Use log differentiation to find derivatives.

Skills:

- Find the derivative of $y = 2^x$.
- Use the log differentiation process to find the derivative of $y = b^x$, where b is a constant.

Examples:

1. Find the derivative of $y = 5^{3x}$.

2. Differentiate $y = \dfrac{e^x \sqrt{x^2+1}}{\left(x^2+2\right)^3}$ by taking the ln of both sides. This is referred to as logarithmic Differentiation.

3. Use logarithmic differentiation to find the derivative of
   a) $y = \left(x^2+1\right)^4 \left(x^3+2x^2\right)^3$
   b) $y = x^{x^2+5}$ (only possible using ln differentiation– not a constant for a base, not a constant for an exponent)