Math 31 **Name** Key

Exponential and Logarithmic Functions Quiz **Date** _____

1. Evaluate $e^{2\ln 3}$ [1]

$= e^{\ln 3^2}$

$= 3^2$

$= 9$

OR

$x = e^{\ln 9}$

$\ln x = \ln 9$

$x = 9$

2. Find the exact solution of the equation $e^{5-2x} = 3$. [1]

$5 - 2x = \ln 3$

$5 - \ln 3 = 2x$

$x = \frac{5 - \ln 3}{2}$

3. Differentiate with respect to x. [2 marks each]

a) $y = e^{4x^5}$

$\frac{dy}{dx} = e^{4x^5}(20x^4)$

a) $\ln y = 4x^5 \ln e$

$\ln y = 4x^5$

$\frac{1}{y}\frac{dy}{dx} = 20x^4$

$\frac{dy}{dx} = [y][20x^4]$

$\frac{dy}{dx} = e^{4x^5}(20x^4)$

b) $y = e^{\tan x^2}$

$\frac{dy}{dx} = e^{\tan x^2}(\sec^2 x^2)(2x)$

c) $y = 5^{2x}$

$$\frac{dy}{dx} = 5^{2x}(\ln 5)(2)$$

$$= (2\ln 5)(5^{2x})$$

d) $y = \dfrac{e^{2x}}{3x+1}$

$f = e^{2x}$ $\quad g = 3x+1$

$$\frac{dy}{dx} = \frac{e^{2x}(2)(3x+1) - e^{2x}(3)}{(3x+1)^2}$$

ooo $6x+2-3$

$$= \frac{e^{2x}(6x-1)}{(3x+1)^2}$$

e) $y = 3x^2 \ln\left(x^2+5x-6\right)$ $\quad f = 3x^2$ $\quad g = \ln(x^2+5x-6)$

$$\frac{dy}{dx} = 6x(\ln(x^2+5x-6)) + 3x^2\left(\frac{1}{x^2+5x-6}\right)(2x+5)$$

$$= 3x\left[2\ln(x^2+5x-6) + \frac{x(2x+5)}{(x+6)(x-1)}\right]$$

f) $y = \ln\left(\frac{4}{\sqrt{2x}}\right)$ $\frac{dy}{dx} = \frac{1}{\frac{4}{\sqrt{2x}}} \frac{d}{dx}\left(4(2x)^{-1/2}\right)$

$$\frac{dy}{dx} = \frac{(2x)^{1/2}}{4}\left[4\left(-\tfrac{1}{2}\right)(2x)^{-3/2}(2)\right]$$

$$\frac{dy}{dx} = \frac{-1}{2x}$$

4. Use logarithmic differentiation to find the derivative of

[3] $y = x^{3x^2}$

$f = 3x^2$ $g = \ln x$

$$\left[\ln y = 3x^2 \ln x\right] \frac{d}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} = 6x \ln x + 3x^2\left(\frac{1}{x}\right)$$

$$\frac{dy}{dx} = x^{3x^2}\left[6x \ln x + 3x\right]$$

$$= \left(x^{3x^2}\right)(3x)(2\ln x + 1)$$

$$= 3x^{3x^2+1}(2\ln x + 1)$$

5. Discuss the curve $y = \ln\left(100 - x^2\right)$ under the following headings.

a) Domain & asymptotes.
b) Intercepts, exact values.
c) Intervals of increase or decrease.
d) Concavity.

[5]

a) domain $(-10, 10)$ asymptotes $x=10$ $x=-10$

b) $0 = \ln(100-x^2)$

$e^0 = 100-x^2$

$x^2 = 99$ $\pm\sqrt{99}$

$x = \pm 3\sqrt{11}$

$y = \ln(100-0)$

$y = \ln 100$

c) $\frac{dy}{dx} = \frac{1}{100-x^2}(-2x) = \frac{-2x}{100-x^2}$ C.N. $x=0$ $x=\pm 10$

	$-2x$	$100-x^2$	$f'(x)$	$f(x)$
$(-10, 0)$	+	+	+	INC
$(0, 10)$	−	+	−	DEC

increasing $(-10, 0)$

decreasing $(0, 10)$

d) $f''(x) = \frac{-2(100-x^2)-(-2x)(-2x)}{(100-x^2)^2} = \frac{-200+2x^2-4x^2}{(100-x^2)^2}$

$= \frac{-200-2x^2}{(100-x^2)^2}$

$= \frac{-2(100+x^2)}{(100-x^2)^2}$

	$-2(100+x^2)$	$(100-x^2)^2$	$f''(x)$
$(-10, 10)$	−	+	−

C.N. $= \pm 10$

$(-10, 10)$ $f''(x) =$ neg $\therefore$ Concave Down on the interval.