MATH 31 Name *Key.*

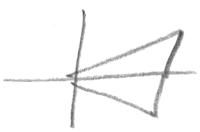
TRIGONOMETRY HW #4 Date ______________

1. Find the local maximum and minimum for $f(x) = x - 2\sin x$ on the interval $[0, 2\pi]$. Justify.

$f'(x) = 1 - 2\cos x$

$0 = 1 - 2\cos x$

$\cos x = 1/2$



Ref $= \frac{\pi}{3}$

$x = \frac{\pi}{3}$ $\quad x = \frac{5\pi}{3}$

$f''(x) = 2\sin x$

$f''\left(\frac{\pi}{3}\right) = 2\sin\frac{\pi}{3} = 2\left(\frac{\sqrt{3}}{2}\right)$

∴ con up — min

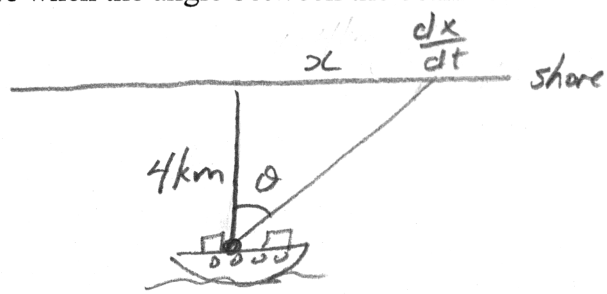
$f''\left(\frac{5\pi}{3}\right) = 2\sin\frac{5\pi}{3} = 2\left(-\frac{\sqrt{3}}{2}\right)$

∴ con down — max

Minimum $f\left(\frac{\pi}{3}\right) = \frac{\pi}{3} - 2\sin\frac{\pi}{3} = \frac{\pi}{3} - 2\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3} - \sqrt{3}$

Maximum $f\left(\frac{5\pi}{3}\right) = \frac{5\pi}{3} - 2\sin\frac{5\pi}{3} = \frac{5\pi}{3} - 2\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{3} + \sqrt{3}$

2. A radar antenna, rotating at 32 rev/min, is located on a ship that is 4 km from a straight shore. How fast does the radar beam sweep across the shore when the angle between the beam and the shore is $\frac{\pi}{4}$?



$\frac{d\theta}{dt} = \frac{32\text{ Rev}}{\text{min}} \times \frac{2\pi}{\text{Rev}} = 64\pi \text{ rad/min}$

$\tan\theta = \frac{x}{4}$

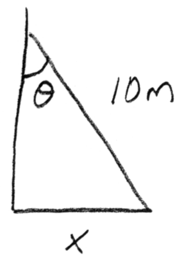
$x = 4\tan\theta$

$\frac{dx}{dt} = 4\sec^2\theta \frac{d\theta}{dt}$ $\quad$ ∴ $\theta = \frac{\pi}{4}$

$\frac{dx}{dt} = 4\left(\frac{\sqrt{2}}{1}\right)^2(64\pi) = 512\pi$

$= 1608 \text{ km/min}$ … 96.5 ⁿ/ₕ

3. A 10 metre long ladder rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 2 m/s, how fast is the angle between the top of the ladder and the wall changing when the angle is $\frac{\pi}{4}$?



$\frac{dx}{dt} = 2$ m/s

$\theta = \frac{\pi}{4}$

$\sin\theta = \frac{x}{10}$

$x = 10\sin\theta$

$\frac{dx}{dt} = 10\cos\theta \frac{d\theta}{dt}$

$2 = 10\left(\cos\frac{\pi}{4}\right)\frac{d\theta}{dt}$

$2 = 10\left(\frac{1}{\sqrt{2}}\right)\frac{d\theta}{dt}$

$\frac{d\theta}{dt} = \frac{2\sqrt{2}}{10}$

$\frac{d\theta}{dt} = \frac{\sqrt{2}}{5}$ rad/s

$\frac{d\theta}{dt} = 0.28$ rad/s "increase"