MATH 31

TRIGONOMETRY HW #1

Name Key.

Date

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1. Evaluate each of the following using trigonometric identities, $\lim_{x\to 0}\frac{\sin x}{x}=1$ and $\lim_{x\to 0}\frac{\cos x-1}{x}=0$.

a) $\lim_{x\to 0}\frac{\sin\frac{1}{2}x}{x}\left(\frac{1/2}{1/2}\right)$

$= \lim_{x\to 0} \frac{1}{2}\frac{\sin\frac{1}{2}x}{\frac{1}{2}x} = \frac{1}{2}\lim_{\frac{1}{2}x\to 0}\frac{\sin\frac{1}{2}x}{\frac{1}{2}x} = \frac{1}{2}(1) = \frac{1}{2}$

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b) $\lim_{x\to 0}\frac{\cos\left(\frac{\pi}{2}-x\right)}{x}$

$\cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$

$= (0)\cos x + (1)\sin x$

$= \sin x$

$= \lim_{x\to 0}\frac{\sin x}{x}$

2

$= 1$

c) $\lim_{x\to 0}\frac{1-\cos 2x}{x}$ OR $\lim_{x\to 0}\frac{-(\cos 2x-1)}{x}\left(\frac{2}{2}\right)$

$= -2\lim_{x\to 0}\frac{\cos 2x-1}{2x}$

$= -2(0)$

$= 0$

$= \lim_{x\to 0}\frac{1-(1-2\sin^2 x)}{x}$

2

$= \lim_{x\to 0}\frac{2\sin^2 x}{x}$

$= 2\lim_{x\to 0}\left(\frac{\sin x}{x}\right)\left(\frac{\sin x}{1}\right) = 2(1)(0)$

$= 0$

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2. Differentiate with respect to x.

a) $y = x^2 \sin x$

$y' = 2x(\sin x) + x^2(\cos x)$

OR

$y' = x(2\sin x + x\cos x)$

(2)

b) $y = \dfrac{\sin x}{1 - 2\cos x}$

$y' = \dfrac{\cos x(1-2\cos x) - \sin x(2\sin x)}{(1-2\cos x)^2}$

$y' = \dfrac{\cos x - 2\cos^2 x - 2\sin^2 x}{(1-2\cos x)^2} = \dfrac{\cos x - 2(\cos^2 x + \sin^2 x)}{(1-2\cos x)^2}$

$y' = \dfrac{\cos x - 2}{(1-2\cos x)^2}$

(2)

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c) $y = \sqrt{x \tan x} = x^{1/2} \tan^{1/2} x$ OR $y = (x\tan x)^{1/2}$

$y' = \frac{1}{2} x^{-1/2} \tan^{1/2} x + x^{1/2}\left[\frac{1}{2}\tan^{-1/2} x\,(\sec^2 x)\right]$

$y' = \frac{1}{2} x^{-1/2} \tan^{-1/2} x\left[\tan x + x\sec^2 x\right]$

$y' = \dfrac{\tan x + x\sec^2 x}{2x^{1/2}\tan^{1/2} x}$

$y' = \dfrac{\tan x + x\sec^2 x}{2\sqrt{x\tan x}}$

$y' = \frac{1}{2}(x\tan x)^{-1/2}\left[1\tan x + x\sec^2\right.$

$y' = \dfrac{\tan x + x\sec^2 x}{2\sqrt{x\tan x}}$

3. Find $\frac{dy}{dx}$ using implicit differentiation for the equation: $\cos(x+y) = y\sin x$

$$\frac{d}{dx}\left[\cos(x+y) = y\sin x\right]$$

$$-\sin(x+y)\left[1+\frac{dy}{dx}\right] = \frac{dy}{dx}\sin x + y\cos x$$

$$-\sin(x+y) - y'\sin(x+y) = y'\sin x + y\cos x$$

$$-\sin(x+y) - y\cos x = y'(\sin x + \sin(x+y))$$

$$y' = \frac{-(\sin(x+y) + y\cos x)}{\sin x + \sin(x+y)}$$

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$y = \sin\frac{\pi}{6} + \cos\frac{\pi}{3}$

4. Find the equation of the tangent line to $y = \sin x + \cos 2x$ when $x = \frac{\pi}{6}$.

$y = \frac{1}{2} + \frac{1}{2} = 1$

$\left(\frac{\pi}{6}, 1\right)$

$$\frac{dy}{dx} = \cos x - \sin 2x\,(2)$$

$$\frac{dy}{dx} = \cos x - 2\sin 2x \quad \text{at } x = \frac{\pi}{6}$$

$$= \cos\frac{\pi}{6} - 2\sin\frac{\pi}{3}$$

$$= \frac{\sqrt{3}}{2} - 2\left(\frac{\sqrt{3}}{2}\right)$$

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$$= -\frac{\sqrt{3}}{2}$$

$$y - 1 = -\frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$$

$$2y - 2 = -\sqrt{3}x + \sqrt{3}\frac{\pi}{6}$$

$$\sqrt{3}x + 2y - 2 - \sqrt{3}\frac{\pi}{6} = 0$$

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