MATH 31

TRIGONOMETRY HW #2

Name ________________

Date ________________

1. Evaluate each of the following using trigonometric identities, $\lim_{x \to 0} \frac{\sin x}{x} = 1$ and $\lim_{x \to 0} \frac{\cos x - 1}{x} = 0$.

a) $\lim_{x \to 0} \frac{\sin^3 2x}{\sin^3 3x}$

$$= \frac{\lim_{2x \to 0} (2x)^3 \left(\frac{\sin 2x}{2x}\right)^3}{\lim_{3x \to 0} (3x)^3 \left(\frac{\sin 3x}{3x}\right)^3}$$

$$= \left(\lim_{x \to 0} \frac{8x^3}{27x^3}\right) \frac{\lim_{2x \to 0} \left(\frac{\sin 2x}{2x}\right)^3}{\lim_{3x \to 0} \left(\frac{\sin 3x}{3x}\right)^3} = \frac{8}{27}\left(\frac{1}{1}\right)^3$$

$$= \frac{8}{27}$$

b) $\lim_{x \to 0} \frac{\cos x - 1}{\sin x} \left(\frac{1/x}{1/x}\right)$

$$= \frac{\lim_{x \to 0} \frac{\cos x - 1}{x}}{\lim_{x \to 0} \frac{\sin x}{x}}$$

$$= \frac{0}{1}$$

$$= 0$$

2. Differentiate with respect to x.

a) $y = \frac{1}{3}\cot 9x$

$$\frac{dy}{dx} = \frac{1}{3}\left[-\csc^2 9x\right](9)$$

$$= -3\csc^2 9x$$

b) $y = \dfrac{x^2}{\cos x}$

$$\frac{dy}{dx} = \frac{2x\cos x - x^2(-\sin x)}{(\cos x)^2}$$

$$= \frac{2x\cos x + x^2 \sin x}{\cos^2 x}$$

OR $$\frac{x(2\cos x + x\sin x)}{\cos^2 x}$$

c) $\tan y = x^2$

$$(\sec^2 y)(y') = 2x$$

$$\frac{dy}{dx} = \frac{2x}{\sec^2 y}$$

$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\csc\frac{\pi}{4} = \sqrt{2}$

$\tan\frac{\pi}{4} = 1$

$\cot\frac{\pi}{4} = 1$

3. Find the equation of the tangent line to $y = \cot^2 x$ when $x = \frac{\pi}{4}$.

$\frac{dy}{dx} = 2\cot x\left(-\csc^2 x\right)$

$\frac{dy}{dx} = -2\cot x \csc^2 x$ at $\frac{\pi}{4}$

$m = -2(1)(\sqrt{2})^2$

$m = -4$

$y = \left(\cot\frac{\pi}{4}\right)^2$

$y = (1)^2 = 1$

$\left(\frac{\pi}{4}, 1\right)$

$y - y_1 = m(x - x_1)$

$y - 1 = -4\left(x - \frac{\pi}{4}\right)$

$y - 1 = -4x + \pi$

$4x + y - 1 - \pi = 0$

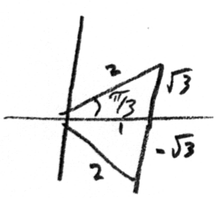
$4x + y - (\pi + 1) = 0$

4. Find the local maximum and minimum for $f(x) = x - 2\sin x$ on the interval $[0, 2\pi]$. Justify using regions of increase and decrease or the second derivative test.

$f'(x) = 1 - 2\cos x$

$0 = 1 - 2\cos x$

$\cos x = \frac{1}{2}$



$x = \frac{\pi}{3}$

$x = \frac{5\pi}{3}$

$f''(x) = 2\sin x$

$f''\left(\frac{\pi}{3}\right) = 2\left(\frac{\sqrt{3}}{2}\right) > 0$, CU $\therefore$ min

$f''\left(\frac{5\pi}{3}\right) = 2\left(-\frac{\sqrt{3}}{2}\right) < 0$, CD $\therefore$ MAX

Minimum $f\left(\frac{\pi}{3}\right) = \frac{\pi}{3} - 2\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\pi}{3} - \sqrt{3}$

Maximum $f\left(\frac{5\pi}{3}\right) = \frac{5\pi}{3} - 2\left(-\frac{\sqrt{3}}{2}\right)$

$= \frac{5\pi}{3} + \sqrt{3}$