MATH 31

Trig HW #3

Name ________________

Date ________________

1. Evaluate. Show at least one step before writing your answer.

a) $\lim_{x\to 0} \frac{\sin x}{\pi x}$ $= \frac{1}{\pi} \lim_{x \to 0} \frac{\sin x}{x}$

$= \frac{1}{\pi}(1)$

$= \frac{1}{\pi}$

b) $\lim_{x\to 0} \frac{\sin^2 3x}{x}$ $= \lim_{x \to 0} 9x \left(\frac{\sin 3x}{3x}\right)^2$

$= 9(0)(1)^2$

6

$= 0$

c) $\lim_{x\to 0} \frac{\sin(\pi - x)}{x}$

$\sin\pi \cos x - \cos\pi \sin x$

$= 0(1) - (-1)\sin x$

$= \sin x$

$= \lim_{x \to 0} \frac{\sin x}{x}$

$= 1$

2. Differentiate with respect to x.

a) $y = 2x^4 \csc x$

$y' = 8x^3 \csc x + 2x^4(-\cot x \csc x)$

$= 2x^3 \csc x\,(4 - x \cot x)$

4

b) $\cos y = \cos 2x$

$(-\sin y)(y') = (-\sin 2x)(2)$

$y' = \dfrac{2 \sin 2x}{\sin y}$

4

3. Find the local maximum and minimum values for $f(x) = \frac{\sqrt{3}}{2}x + \cos x$, $[0, 2\pi]$. Justify using regions of increase and decrease or the second derivative test.

$f'(x) = \dfrac{\sqrt{3}}{2} - \sin x$

$0 = \dfrac{\sqrt{3}}{2} - \sin x$

$\sin x = \dfrac{\sqrt{3}}{2}$

$x = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$

$f''(x) = -\cos x$

$f''\left(\frac{\pi}{3}\right) = -\cos\frac{\pi}{3} = -\frac{1}{2}$ CD ∴ max

$f\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}\left(\frac{\pi}{3}\right) + \cos\frac{\pi}{3}$

$= \frac{\sqrt{3}\pi}{6} + \frac{1}{2} \rightarrow \frac{3 + \sqrt{3}\pi}{6}$ MAX

$f''\left(\frac{2\pi}{3}\right) = -\cos\left(\frac{2\pi}{3}\right) = \frac{1}{2}$ CU ∴ min

$f\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}\left(\frac{2\pi}{3}\right) + \cos\frac{2\pi}{3}$

$= \frac{\sqrt{3}\pi}{3} - \frac{1}{2} \rightarrow \frac{-3 + 2\sqrt{3}\pi}{6}$