Review: Reciprocal, Quotient, and Pythagorean Identities

Outcome: Review the reciprocal, Quotient and Pythagorean Identities

Warm up:

- In calculus, it is often helpful to change trigonometric expressions from one form to an equivalent form that is easier to work with.
- An equation that equates the two equivalent trigonometric expressions is called a trigonometric identity.
- *Note: The measure of angles in this module will be radian measure, it is required for the simplification of the derivatives of trigonometric functions.*

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

The quotient identities are as follows:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

The advantage of the reciprocal and quotient identities is they allow you to rewrite any of the other four ratios in terms of sine and cosine.

Examples:

1. Simplify $(\cos^2\theta)(\sec\theta)(\tan\theta)$.

2. Simplify $\frac{\cot\theta\cos\theta}{\csc\theta}$

3. Simplify $(\sin^2\theta)(\csc\theta)(\cot\theta)$

The Pythagorean Identities

The basic Pythagorean identity is: $\sin^2 x + \cos^2 x = 1$.

There are three identities that make up all the **Pythagorean identities** and are used often in calculus problems that involve trigonometry. To summarize, the identities are as follows:

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Problems that involve changing trigonometric expressions from one form to another are written as equations. Your task is to transform one side of the equation into the other. Although an equal sign is present, the rules for equations (adding or multiplying both sides by the same amount) are **not** used. The expressions are transformed individually.

To accomplish the transformation of one or the other or both expressions, follow these steps:

Step 1: Replace trigonometric ratios with equivalent expressions that contain $\sin\theta$ and $\cos\theta$

Step 2: Look for forms of the Pythagorean identity that can be replaced.

Step 3: Manipulate the expressions algebraically. Factoring, creating common denominators, and multiplying by an expression equivalent to 1 are the most common.

Step 4: Be inventive. There is more than one correct way to show that two expressions are equivalent. If one thing doesn't work, try another. With practice, these transformations will become familiar and you will accomplish them with ease.

4. Prove: $\sin x \tan x = \sec x - \cos x$.

5. Prove: $\dfrac{\cos^2\theta}{1+\sin\theta} = 1 - \sin\theta$.

6. Prove: $\dfrac{\sin x}{1+\cos x} = \dfrac{1-\cos x}{\sin x}$.

7. Prove: $\dfrac{1+\tan\theta}{1+\cot\theta} = \dfrac{1-\tan\theta}{\cot\theta - 1}$.

EXERCISE 6.4

B *The following identities involve the reciprocal, quotient, and Pythagorean relationships. Prove each one.*

1. $\sin x \tan x = \sec x - \cos x$
2. $\cos^4 x - \sin^4 x = 1 - 2\sin^2 x$
3. $\csc^2 x + \sec^2 x = \csc^2 x \sec^2 x$
4. $\cos^2 x \cos^2 y + \sin^2 x \sin^2 y + \sin^2 x \cos^2 y + \sin^2 y \cos^2 x = 1$
5. $\sec^2 x - \sec^2 y = \tan^2 x - \tan^2 y$
6. $\dfrac{\tan x + \tan y}{\cot x + \cot y} = (\tan x)(\tan y)$
7. $(\sec x - \cos x)(\csc x - \sin x) = \dfrac{\tan x}{1 + \tan^2 x}$
8. $\cos^6 x + \sin^6 x = 1 - 3\sin^2 x + 3\sin^4 x$
9. $\sec^6 x - \tan^6 x = 1 + 3\tan^2 x \sec^2 x$

Review: Solving Trigonometric Equations

Outcomes: Solve trigonometric equations that have exact solutions.

Use triangles or the unit circle to find exact solutions to trigonometric equations:

State the exact values of:

Cos $\frac{\pi}{6}$ = Cos $\frac{\pi}{3}$ = Cos $\frac{\pi}{4}$ =

Sin $\frac{\pi}{6}$ = Sin $\frac{\pi}{3}$ = Sin $\frac{\pi}{4}$ =

Tan $\frac{\pi}{6}$ = Tan $\frac{\pi}{3}$ = Tan $\frac{\pi}{4}$ =

Examples:

1. Find the exact value(s) of θ where $0^0 \leq \theta < 360^0$

a) $\sin\theta = \frac{1}{\sqrt{2}}$ b) $\cos\theta = \frac{\sqrt{2}}{2}$

c) $\sec\theta = -2$ d) $\cot\theta = 1$

e) $\sin\theta = 0$ f) $\cos\theta = -1$

2. Solve the equation $\cos^2 x - \cos x - 2 = 0$ for $0 \leq x < 2\pi$. Then write the general solution of the equation.

3. Find the solutions for $\cos^2 x + 2\sin x - 2 = 0$, where $0 \le x < 2\pi$ Then write the general solution of the equation.

4. Solve $\cos 3x = -\frac{\sqrt{2}}{2}$, where $0 \le x < 2\pi$

5. If $\sec 2x + \dfrac{1}{\cos x} = 0$ where $0 \le x < \pi$, solve for *x*.

6. Find the solutions for $\cos x = 1 + \sin x$ where $0 \le x < 2\pi$

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1. Solve for x in the interval $[0, 2\pi]$.

(a) $\sin x = \dfrac{\sqrt{3}}{2}$ (b) $\cos x = \frac{1}{2}$ (c) $\tan x = -1$

(d) $\sec x = -2$ (e) $\sin x = -\frac{1}{2}$ (f) $\cos^2 x = \frac{1}{4}$

3. Solve for x in the given interval.

(a) $\sin x - \sin x \tan x = 0, [0, \pi]$

(b) $\sin x \tan 3x = 0, [-\pi, 0]$

(c) $6 \sin^2 x - 5 \cos x - 2 = 0, [0, 2\pi]$

4. Solve for x.

(a) $\cos 2x = \cos^2 x, -\pi \leqslant x \leqslant \pi$

(b) $\sin 2x = \cos x, -\pi \leqslant 2x \leqslant \pi$

(c) $\cos^2 x - 2 \sin x \cos x - \sin^2 x = 0, 0 \leqslant 2x \leqslant \pi$

(d) $\tan 2x = 8 \cos^2 x - \cot x, 0 \leqslant x \leqslant \dfrac{\pi}{2}$

(e) $\tan x + \sec 2x = 1, -\dfrac{\pi}{2} \leqslant x \leqslant \dfrac{\pi}{2}$