The Quotient Rule and the Chain Rule

$P(x) = fg$ then $P'(x) = f'g + fg'$

$Q(x) = \dfrac{f}{g}$ then $Q'(x) = \dfrac{f'g - fg'}{g^2}$

Use the quotient rule to find the derivative of: $f(x) = \dfrac{x^5}{x^2}$ $\rightarrow$ $f(x) = x^3$

$f'(x) = 3x^2$

$f = x^5$ $\quad$ $g = x^2$

$f' = 5x^4$ $\quad$ $g' = 2x$

$$f'(x) = \frac{f'g - fg'}{g^2}$$

$$f'(x) = \frac{(5x^4)(x^2) - (x^5)(2x)}{[x^2]^2}$$

$$f'(x) = \frac{5x^6 - 2x^6}{x^4} = \frac{3x^6}{x^4} = 3x^2$$

Differentiate $F(x) = \dfrac{x^2 + 2x - 3}{x^3 + 1}$

$f = x^2 + 2x - 3$ $\quad$ $g = x^3 + 1$

$f' = 2x + 2$ $\quad$ $g' = 3x^2$

$$F'(x) = \frac{f'g - fg'}{g^2}$$

$$F'(x) = \frac{(2x+2)(x^3+1) - (x^2+2x-3)(3x^2)}{[x^3+1]^2}$$

$$F'(x) = \frac{2x^4 + 2x + 2x^3 + 2 - 3x^4 - 6x^3 + 9x^2}{(x^3+1)^2}$$

$$F'(x) = \frac{-1x^4 - 4x^3 + 9x^2 + 2x + 2}{(x^3+1)^2}$$

2.3 Quotient and Chain Intro

Review factoring and algebra skills to simplify the following:

1. $6mx^2 + 12my$

$= 6m(x^2 + 2y)$

2. $3a(x^2 + a) + 6a^2(x + 1)$

$= 3a[1(x^2 + a) + 2a(x + 1)]$

$= 3a(x^2 + 2ax + 3a)$

3. $8x^2(x^4 + 5) + 4x^3(x^3 - 2)$

$4x^2(2(x^4 + 5) + x(x^3 - 2))$

$4x^2(3x^4 - 2x + 10)$

4. $3x(x^2 + 2x + 3) - 3x^2(x + 4)$

$= 3x[(x^2 + 2x + 3) - x(x + 4)]$

$= 3x(-2x + 3)$

$\frac{x^{\frac{1}{2}}}{x^{-1/2}}$

5. $x^{\frac{-1}{2}}(2x^2 + 1) + x^{\frac{1}{2}}(x - 3)$

$= x^{-1/2}[1(2x^2 + 1) + x(x - 3)]$

6. $\frac{2}{3}x^{\frac{-1}{3}}(x^2 + 2) + 2x^{\frac{2}{3}}(x + 1)$

$= \frac{2}{3}x^{-1/3}[1(x^2 + 2) + 3x(x + 1)]$

2.3 Quotient and Chain Intro

Find $\frac{dy}{dx}$ if $y=\frac{2\sqrt{x}}{2+3x}$

$f = 2x^{1/2}$ $\quad g = 2+3x$

$f' = x^{-1/2}$ $\quad g' = 3$

$$\frac{dy}{dx} = \frac{f'g - fg'}{g^2}$$

$$\frac{dy}{dx} = \frac{x^{-1/2}(2+3x) - 2x^{1/2}(3)}{[2+3x]^2}$$

$$\frac{dy}{dx} = \frac{x^{-1/2}[1(2+3x) - 2x(3)]}{(3x+2)^2}$$

$$\frac{dy}{dx} = \frac{2-3x}{\sqrt{x}(3x+2)^2}$$

Review the product rule:

Find the derivative of:

a) $f(x) = (5x+2)(5x+2)$

b) $y = (3x^2+1)(3x^2+1)$

c) $g(x) = (2x+3)^3$

a) $f = 5x+2$ $\quad g = 5x+2$

$f' = 5$ $\quad g' = 5$

$f'(x) = f'g + fg'$

$f'(x) = 5(5x+2) + 5(5x+2)$

$= 10(5x+2)$

b) $\frac{dy}{dx} = 12x(3x^2+1)$

c) $f = 2x+3$ $\quad g = (2x+3)(2x+3)$

$f' = 2$

$f = 2x+3$ $\quad g = 2x+3$

$f' = 2$ $\quad g' = 2$

$g' = 2(2x+3) + 2(2x+3)$

$g' = 4(2x+3)$

$g'(x) = f'g + fg'$

$g'(x) = 2(2x+3)^2 + (2x+3)(4)(2x+3)$

$= 2(2x+3)^2 + 4(2x+3)^2$

$= 6(2x+3)^2$

Can you see a possible solution to finding $\frac{dy}{dx}$ without using the product rule?

a) $f(x)=(5x+2)^2$

b) $y=(3x^2+1)^2$

c) $g(x)=(2x+3)^3$

c) $g(x)=(2x+3)^{\boxed{3}}$ ... $g'(x)=6(2x+3)^2$

$g'(x)=3(2x+3)^2\frac{d}{dx}(2x+3)$

$g'(x)=3(2x+3)^2(2)$

$=6(2x+3)^2$

a) $f(x)=(5x+2)^2$

$f'(x)=2(5x+2)^1\cdot\frac{d}{dx}(5x+2)$

$f'(x)=2(5x+2)(5)$

$f'(x)=10(5x+2)$

CHAIN RULE: $y=f(x)^n$ *then* $\frac{dy}{dx}=n[f(x)]^{n-1}\cdot\frac{d}{dx}[f(x)]$

2.3 Quotient and Chain Intro

Find the derivative of the following.

a) $y=(x^3+1)^{\boxed{3}}$
b) $f(x)=\sqrt{3x+4}$
c) $y=\left(3x^3-x^{\frac{3}{2}}+2\right)^{-\frac{5}{2}}$
d) $y=(x^2-x+2)^8$
e) $f(x)=\dfrac{1}{\sqrt[3]{1-x^4}}$

a) $\frac{dy}{dx}=3(x^3+1)^2\frac{d}{dx}(x^3+1)$

$\frac{dy}{dx}=3(x^3+1)^2(3x^2)$

$=9x^2(x^3+1)^2$

b) $f(x)=(3x+4)^{1/2}$

$f'(x)=\frac{1}{2}(3x+4)^{-1/2}\cdot\frac{d}{dx}(3x+4)$

$f'(x)=\dfrac{3}{2\sqrt{3x+4}}$

e) $f(x)=(1-x^4)^{-1/3}$

$f'(x)=-\frac{1}{3}(1-x^4)^{-\frac{4}{3}}\frac{d}{dx}(1-x^4)$

$=-\frac{1}{3}(1-x^4)^{-4/3}(-4x^3)$

$=\dfrac{4x^3}{3(1-x^4)^{4/3}}$

$=\dfrac{4x^3}{3\sqrt[3]{(1-x^4)^4}}$