Math 31

Exponential and Logarithmic Functions Quiz

Name Key

Date ________

1. Evaluate $e^{\ln 8}$ $= a$

$\log_e a = \ln 8$

[1] $a = 8$

2. Find the exact solution to the equation: $e^{5+2x} = 7$.

$5 + 2x = \ln 7$

[2] $2x = \ln 7 - 5$

$x = \dfrac{\ln 7 - 5}{2}$

3. Differentiate with respect to x. [2 marks each]

a) $y = e^{5x^2}$

$\dfrac{dy}{dx} = e^{5x^2} \cdot 10x$

$\boxed{\text{OR}}$ $= 10x e^{5x^2}$

b) $y = 2^{x^3}$

$\dfrac{dy}{dx} = (2^{x^3})(\ln 2)(3x^2)$

$\boxed{\text{OR}}$ $= (3x^2 \ln 2)(2^{x^3})$

b) $\left[\ln y = x^3 \ln 2\right] \dfrac{d}{dx}$

$\left[\dfrac{1}{y}\right] \dfrac{dy}{dx} = 3x^2 \ln 2$

$\dfrac{dy}{dx} = (2^{x^3})(3x^2 \ln 2)$

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4. Differentiate with respect to x. [3 marks each]

a) $y = e^{\sin^2(5x)}$

$$\frac{dy}{dx} = e^{\sin^2(5x)}\left[2\sin(5x)\cdot\cos(5x)\cdot 5\right]$$

$$= 5\sin(10x)e^{\sin^2(5x)}$$

$\ln y = \ln e^{\sin^2 5x}$

$\left[\ln y = \sin^2 5x\right]\frac{d}{dx}$

$\frac{1}{y}\cdot\frac{dy}{dx} = 2\sin 5x\cos 5x\,(5)$

$\frac{dy}{dx} = e^{\sin^2(5x)}(5)(\sin 10x)$

b) $y = ln\left(\frac{2}{\sqrt{x}}\right)$ OR $y = \frac{4}{\sqrt[x]{5}} = 4\cdot 5^{-1/x}$

$$\frac{dy}{dx} = \frac{\sqrt{x}}{2}\cdot\frac{d}{dx}\left(2x^{-1/2}\right)$$

$$= \frac{\sqrt{x}}{2}\cdot\left[-1x^{-3/2}\right]$$

$$= \frac{x^{1/2}}{2}\left(\frac{-1}{x^{3/2}}\right)$$

$$= -\frac{1}{2x}$$

$\left[y = \ln 2 - \frac{1}{2}\ln x\right]\frac{d}{dx}$

$\frac{dy}{dx} = -\frac{1}{2}\left(\frac{1}{x}\right) = -\frac{1}{2x}$

$$\frac{dy}{dx} = \left[\frac{4}{\sqrt[x]{5}}\right]\cdot\ln 5\cdot\left[-1x^{-1}\right]\frac{d}{dx}$$

$$= \frac{4}{\sqrt[x]{5}}\cdot\ln 5\cdot(x^{-2})$$

$$= \frac{4\ln 5}{\sqrt[x]{5}\,x^2}$$

$\left[y = \frac{4}{5^{1/x}}\right]\ln$

$\left[\ln y = \ln 4 - x^{-1}\ln 5\right]\frac{d}{dx}$

$\left[\frac{1}{y}\right]\left[\frac{dy}{dx}\right] = x^{-2}\ln 5$ $\therefore \frac{dy}{dx} = \frac{4}{\sqrt[x]{5}}\left(\frac{\ln 5}{x^2}\right)$

4. Differentiate with respect to x. [3 marks each]

c) $y = x^2 e^x \sqrt{x^3 + x^2 + 5}$

$$\ln y = \ln x^2 + \ln e^x + \frac{1}{2}\ln(x^3 + x^2 + 5)$$

$$\frac{d}{dx}\left[\ln y = \ln x^2 + x + \frac{1}{2}\ln(x^3 + x^2 + 5)\right]$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2}(2x) + 1 + \frac{1}{2}\left(\frac{1}{x^3 + x^2 + 5}\right)(3x^2 + 2x)$$

$$\frac{dy}{dx} = [y]\left[\frac{2}{x} + 1 + \frac{x(3x+2)}{2(x^3 + x^2 + 5)}\right]$$

$$\frac{dy}{dx} = \left[x^2 e^x \sqrt{x^3 + x^2 + 5}\right]\left[\frac{2(2)(x^3+x^2+5) + 2x(x^3+x^2+5) + x^2(3x+2)}{2x(x^3 + x^2 + 5)}\right]$$

$$= \frac{x^2 e^x \sqrt{x^3 + x^2 + 5}\,(4x^3 + 4x^2 + 20 + 2x^4 + 2x^3 + 10x + 3x^3 + 2x^2)}{2x(x^3 + x^2 + 5)}$$

$$= \frac{x e^x (2x^4 + 9x^3 + 6x^2 + 10x + 20)}{2\sqrt{x^3 + x^2 + 5}}$$

5. Given the curve $y = \ln(36 - x^2)$

a) Find the exact x-intercepts and y-intercept.

b) Use derivatives to determine:

- Intervals of increase and/or decrease.
- ~~Regions of Concavity.~~

[4]

a) $x = 0$

$y = \ln(36 - 0)$

$y = \ln 36$

$y = 0$

$0 = \log_e(36 - x^2)$ OR $e^0 = 36 - x^2$

$1 = 36 - x^2$

$x = \pm\sqrt{35}$

b) $\frac{d}{dx}\left[y = \ln(36 - x^2)\right]$

$\frac{dy}{dx} = \frac{1}{36 - x^2}(-2x)$

$0 = \frac{-2x}{36 - x^2}$

CN $x = 0$ $x = \pm 6$

	$-2x$	$36 - x^2$	$\frac{dy}{dx}$	y
$(-6, 0)$	+	+	+	INC
$(0, 6)$	−	+	−	DEC