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Math 31 Name Key

Trig Limits and Derivatives Quiz – 2019

1. Evaluate each limit. Justify your answers with algebraic processes. [3 marks each]

a) $\lim\limits_{x\to 0}\frac{\sin\left(\frac{1}{2}x\right)}{2x}$

b) $\lim\limits_{x\to 0}\frac{1-\cos x}{x\sin x}$

c) $\lim\limits_{x\to 0}\frac{2\tan^2 x}{x^2}$

a) $\lim \frac{\sin(\frac{1}{2}x)\,(\frac{1}{2}x)}{2x\,(\frac{1}{2}x)}$

$= \lim\limits_{x\to 0} \frac{\sin(\frac{1}{2}x)}{(\frac{1}{2}x)} \cdot \frac{1}{4}$

$= (1)(\frac{1}{4})$

$= \frac{1}{4}$

c) $\lim\limits_{x\to 0} \frac{\frac{2\sin^2 x}{\cos^2 x}}{x^2}$

$= \lim\limits_{x\to 0} \frac{2\sin^2 x}{x^2} \cdot \frac{1}{\cos^2 x}$

$= 2(1)^2\left(\frac{1}{1}\right)^2$

$= 2$

b) $\lim\limits_{x\to 0} \frac{1-\cos x}{x\sin x}\left(\frac{1+\cos x}{1+\cos x}\right)$

$= \lim\limits_{x\to 0} \frac{1-\cos^2 x}{(x)(\sin x)(1+\cos x)}$

$= \lim\limits_{x\to 0} \frac{\sin^2 x}{(x)(\sin x)(1+\cos x)}$

$= \lim\limits_{x\to 0} \left(\frac{\sin x}{x}\right)\left(\frac{\sin x}{\sin x}\right)\left(\frac{1}{1+\cos x}\right)$

$= (1)(1)\left(\frac{1}{1+1}\right)$

$= \frac{1}{2}$

OR

$\lim\limits_{x\to 0} \frac{1-\cos x}{x(\sin x)}\left(\frac{\sin x}{\sin x}\right)$

$= \lim\limits_{x\to 0} \left(\frac{\sin x}{x}\right)\left(\frac{1-\cos x}{\sin^2 x}\right)$ $1-\cos^2 x$

$= \lim\limits_{x\to 0} \left(\frac{\sin x}{x}\right)\left(\frac{1-\cos x}{(1-\cos x)(1+\cos x)}\right)$

$= \lim\limits_{x\to 0} \left(\frac{\sin x}{x}\right)\left(\frac{1}{1+\cos x}\right)$

$= (1)\left(\frac{1}{1+1}\right)$

$= \frac{1}{2}$

2. Differentiate with respect to x; solve for $\frac{dy}{dx}$. Be sure to simplify your answers; use the best trig ratios possible. [3 marks each]

a) $y = sin^2(5x)$

b) $y = \frac{sinx}{1-2cosx}$

c) $tany = x^2cosx$

2a) $\frac{dy}{dx} = 2\sin(5x)[\cos(5x)][5]$ ... $2\sin A\cos A = \sin(2A)$

$= [\sin(2\cdot 5x)][5]$

$= 5\sin(10x)$

b) $\frac{dy}{dx} = \frac{(\cos x)(1-2\cos x) - \sin x(2\sin x)}{[1-2\cos x]^2}$

$= \frac{\cos x - 2\cos^2 x - 2\sin^2 x}{[1-2\cos x]^2}$ ... $-2(\cos^2 x + \sin^2 x)$

$= \frac{\cos x - 2}{(1-2\cos x)^2}$

c) $(\sec^2 y)\frac{dy}{dx} = 2x\cos x + x^2(-\sin x)$

$\frac{dy}{dx} = \frac{2x\cos x - x^2\sin x}{\sec^2 y}$ OR $\frac{x(2\cos x - x\sin x)}{\sec^2 y}$

3. Determine the interval where the curve $y = sin^2x + 2cosx,\ 0 \leq \theta < 2\pi$ is concave up. [4 marks]

$$\frac{dy}{dx} = 2\sin x\cos x - 2\sin x$$

$$y'' = 2(\cos x\cos x + \sin x(-\sin x)) - 2\cos x$$

$$0 = 2(\cos^2 x - \sin^2 x) - 2\cos x$$

$$0 = 2(\cos^2 x - (1-\cos^2 x)) - 2\cos x$$

$$0 = 2(2\cos^2 x - 1) - 2\cos x$$

$$0 = 4\cos^2 x - 2\cos x - 2$$

$$0 = 2\cos^2 x - \cos x - 1$$

$$0 = (2\cos x + 1)(\cos x - 1)$$

$\cos x = -\frac{1}{2}$ $\qquad$ $\cos x = 1$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$ $\qquad$ $x = 0, 2\pi$

	$2\cos x + 1$	$\cos x - 1$	$f''(x)$	$f(x)$
$(0, \frac{2\pi}{3})$	+	−	−	CD
$(\frac{2\pi}{3}, \frac{4\pi}{3})$	−	−	+	CU
$(\frac{4\pi}{3}, 2\pi)$	+	−	−	CD

Concave up: $(\frac{2\pi}{3}, \frac{4\pi}{3})$